Effects of faults in cut-off walls

Etude des défauts dans les écrans étanchés

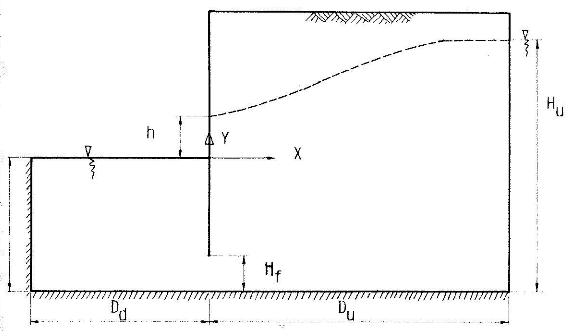
A.HOLEYMAN, Ir., Dr., Manager of the Research and Geotechnical Department, Franki S.A., Belgium, part time Assistant Professor at the Université Libre de Bruxelles, Belgium
M.BOTTIAU, Ir., Project Engineer, Research and Geotechnical Department, Franki S.A., Belgium
J.NUYENS, Ir., Dr., Professor at the Université Libre de Bruxelles, Belgium, Director of the Laboratory Jacques Verdeyen, Bruxelles, Belgium

SYNOPSIS : Seepage resulting from faults in cut-off walls is studied. Cut-off walls are installed into porous media to control underseepage. The analysis is carried out in a two dimensional approach and for a steady flow. Several tools are used involving both analytical and numerical methods. The most noteworthy results are summarized by charts.

INTRODUCTION

Underground water management often calls for the installation of cut-off walls in porous media.

Flow can result from the presence of faults in these cut-off walls. In this paper the fault is defined as the gap between the bottom of the wall and the horizontal impermeable boundary. The wall is considered as impermeable and infinitely thin. The characteristics of the flow depend on the soil permeability and on the geometrical parameters of the seepage region. These parameters are outlined in fig. 1 illustrating the geometry of the seepage region. This figure points out the originality of the boundary conditions by comparison with the already treated problem of cut-off walls under dams. In the case considered here, only the downstream condition is governed by a constant head. On the other hand, the upstream condition is a given head H_u at the upstream distance D_u. This implies the existence of a free surface which considerably complicates the study. From the study of the influence of the parameters, it will be shown that the major resulting features of the seepage are the wall head loss h and the seepage flow Q.



H_d : downstream head
H_u : upstream head
D_d : downstream distance
D_u : upstream distance
H_f : height of the fault
h : wall head loss

Fig. 1. Geometry of the seepage region

METHODS

A general study was carried out. Both analytical and numerical methods were used :

- analytical methods : Conformal Mapping; Sum of Potentials;
- Numerical methods : Finite Element Method, (FEM), Boundary Element Method (BEM).

ANALYTICAL METHODS

Conformal Mapping

Conformal Mapping allows, by mean of an analytic function, to transform a complex area where a solution of Laplace's equation is sought into a simpler one where the solution is easier to find. One re-transforms then it back to the real domain.

Conformal Mapping of the exact problem led us to useless integral expressions like expression (0) below.

The flow coordinate $z = x + iy$ is expressed with respect to an auxiliary coordinate

$\zeta = \xi + i\eta$

$\zeta = sn^2 (\frac{\omega}{C}, \lambda)$

where sn : elliptic function of the first kind
ω : $\phi + i\,\psi$ and $C = H_u/K$ with
ϕ : seepage potential
ψ : seepage flow function
K : elliptic integral modulus
λ : inverse elliptic integral modulus $\sqrt{1-K^2}$

$$z = \frac{i}{F(\zeta_1) - F(\zeta_2)} \cdot \frac{H_u}{2K} \cdot \int_o^\zeta [F(\zeta 1) - F(\zeta)] \cdot \frac{d\zeta}{\sqrt{(1-\zeta)\cdot(1-\lambda_2^2\,\zeta)\cdot\zeta}} + i\int_o^\zeta \frac{H_u}{2K} \cdot \frac{\zeta}{\sqrt{\zeta\cdot(1-\zeta)\cdot(1-\lambda^2\zeta)}} + D_u \quad (0)$$

where F can be written :

$$F(\zeta) = \int_o^\zeta \frac{(\zeta - \zeta_3)}{\sqrt{(\zeta-\zeta_1)\,(\zeta - \frac{1}{\lambda^2})^3 \cdot \zeta^3}}\, d\zeta$$

Four equations are known from the boundary conditions which can theoretically enable us to determine the four unknowns $\zeta_1, \zeta_2, \zeta_3, \lambda$.

Despite efforts and time, no exact solution could be found. (The equations were too complex to be conveniently treated as to give analytical results).

Therefore, a simplified area with an infinite half-space was treated. The solution is shown on fig. 2. Unfortunately, a straightforward link between this seepage and the real one could not be found.

Sum of Potentials

The well-known principle of superposition of potentials consists summing elementary potential functions to get the potential in a complex domain. This principle was extended to the sum of drainage trenches, starting from the assumption that the fault in the axis of the cut-off wall is an equipotential line.

Once again, this analytical process gave no exploitable result. The free surface, for example, was impossible to model.

Conclusion From The Analytical Study.

We can thus conclude, as far as the analytical methods are concerned, that the obtained results were far from satisfactory: the resulting analytical expressions are too complicated or result from excessive simplifications to have a practical use. However, they have outlined some very useful features of the work and have enabled us to find hypotheses for simpler ways of investigation.

NUMERICAL METHODS

Modellization

A detailed finite element study was performed at the Continum Mechanics Department, Université Libre de Bruxelles, Belgium.

For two cases treated by the FEM, the potential field was also assessed by a Boundary Element program developed at the Université de Liège, Belgium. The agreement between the results obtained by both methods fully reinforces the validity of these results.

For the Finite Element Study, the succession of operations is the following.

A linear grid is automatically generated determining a geometry for which the potential field is assessed using the FEM theory. A convergence test is then applied on the free surface in order to iteratively approximate to the real free surface. We verify therefore a condition $\phi = z$. (ϕ = potential; z = water head). A typical linear grid is shown on fig. 3. Fig. 4 illustrates typical results. Three points must be noted :

- A non-seepage corner does exist at the upstream contact of the wall.
- The vertical line through the fault is confirmed to be an equipotential line, as it was assumed for the sum of potentials.
- The flow lines rapidly become subhorizontal, which allows us to assume a very low gradient $\frac{\partial\phi}{\partial y}$.

Simplified Expressions

Below are developped expressions based on the observations previously mentioned in order to intrepret the results obtained by

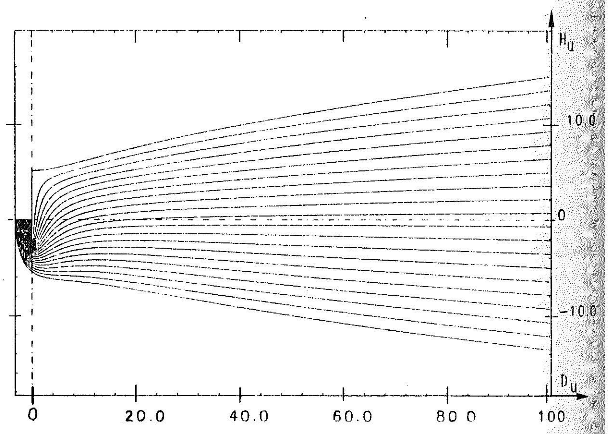


Fig. 2. Seepage for an infinite half-space.

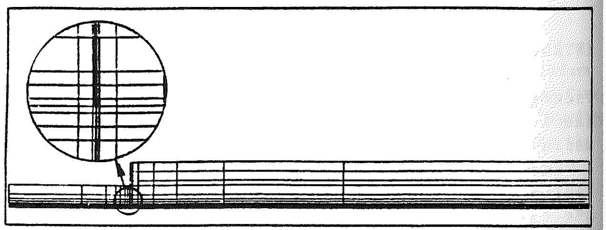


Fig. 3. Typical FEM grid.

the FEM. The notations used are those of fig. 1.

Darcy's law is written

$$v = - Kj = - K \frac{d\phi}{ds}$$

where : v is the seepage velocity
j is the seepage gradient
K is the permeability coefficient.
ϕ is the seepage potential
s is the flow coordinate

The linear seepage flow per current meter of the wall Q $[m^3\ s^{-1}/m]$ can be expressed by

$$Q = \int_{t=o}^{t=H_d+z} v\, dt$$

where t is the potential coordinate.
The reduced linear flow is defined as Q/K.
Two hypotheses are then made :

1/ $\frac{d\phi}{ds}$ has a constant value along a vertical line at a sufficient distance upstream of the wall

2/ the vertical gradient $\frac{\partial\phi}{\partial y}$ has a low value so that we can write $\frac{d\phi}{ds} = \frac{\partial\phi}{\partial x}$

These hypotheses are in fact similar to those made by Dupuits (1794) for the study of wells.

We can then write

$$\frac{Q}{K} = \frac{d\phi}{ds}(H_d + z) = \frac{\partial\phi}{\partial x}(H_d + z)$$

And, as $\phi = z$ for a point on the free surface

$$\frac{Q}{K}\, dx = (H_d + z)\, dz = (H_d + z)\, d(H_d + z)$$

Which can be readily intergrated into :

$$\frac{Q}{K}\, x = \frac{(H_d + z)^2}{2} - \frac{(H_d + h)^2}{2} \qquad (1)$$

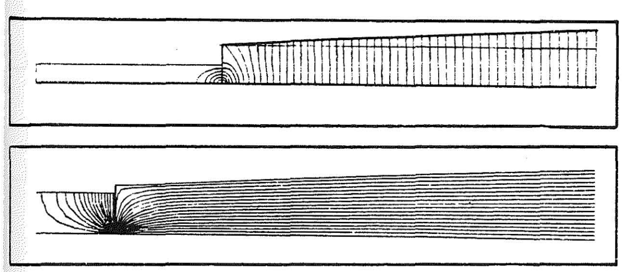


Fig. 4. Typical results obtained with the FEM

RESULTS

From equation (3), we can obtain several expressions.
The most significant are sumarized below.
It must be previously noted that the influence of the downstream distance D_d can be neglected, as long as it is not less than 2 H_d.

Seepage Flow With Respect To The Piezometric Head

If $x = D_u$, $z = H_u - H_d$ and we can write, with equation (3)

$$2 \cdot \frac{Q}{K} D_u = H_u^2 - (H_d + h)^2 \qquad (4)$$

This fundamental equation establishes the link between the two main results of our study : the seepage flow Q and the wall head loss h. Two measurements of common practice (h and H_d) will therefrom enable us to determine the reduced flow Q/K of the seepage.
Fig. 5 shows how the numerical results fit the theoretical law. We can then write the real law as

$$2 D_u \frac{Q}{K} = \alpha \cdot (H_u^2 - (H_d + h)^2)$$
with $\alpha = f(H_d)$

Seepage Flow With Respect To Upstream Distance

Starting from equations (3) and (4)

$$\frac{Q}{K} x = \frac{(H_d + z)^2}{2} - \frac{(H_d + h)^2}{2}$$

$$\frac{Q}{K} D_u = \frac{H_u^2}{2} - \frac{(H_d + h)^2}{2}$$

We write

$$\frac{Q}{K}(x - D_u) = \frac{(H_d + z)^2}{2} - H_u^2 \qquad (5)$$

Fig. 6 shows the free surface obtained with equation (5) and the real free surface for the same case. This figure confirms the full validity of this law.

Reference Downstream Head/Reduction Ratio

If all the dimensions of the seepage domain are multiplyed by a factor p; the seepage gradient remains the same $\frac{d\phi}{ds} = \frac{d\ (p\phi)}{d\ (ps)}$ and the section z x 1 m^2 is multiplied by p.
The resulting seepage flow is consequently also multiplied by p.
One of the seepage dimensions can therefore be chosen as a reference value.
The downstream head was chosen as reference dimension. We then determine the reduction ratio p defined as the ratio between the real one.

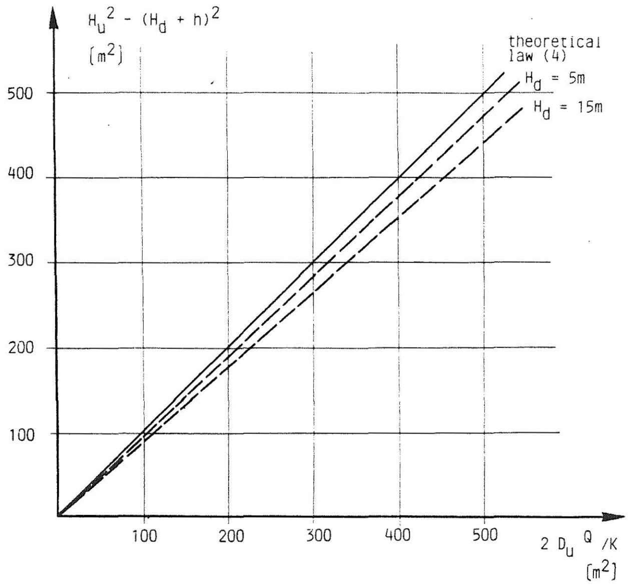


Fig. 5. Seepage flow with respect to the piezometric head : theoretical and real law

The other dimensions of the domain are multiplied by this reduction ratio p in order to define a new domain. The flow assessed with this domain of seepage has finally to be multiplied by p to assess the real flow.
For this reason, all the assessments of our study were made with a reference downstream head of 5 m and non-dimensional ratios with respect to this downstream head were used to present the results.

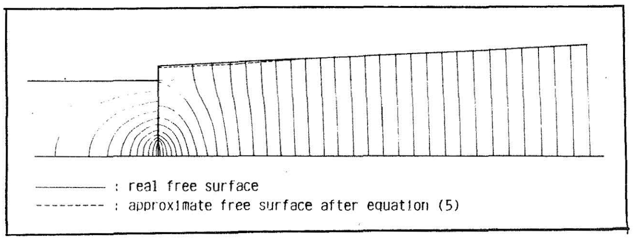


Fig. 6 Agreement between the real and the approximate free surface

CHARTS

As previously mentioned, a detailed study was carried out, assessing the flow Q and the wall head loss h for a lot of geometries of the seepage flow.
The results were then extrapolated by means of the expressions explained above.
Charts have been developed to provide significant results, using the following non-dimensional parameters :

adimensional seepage flow	$q = \frac{Q}{K\ H_d}$
adimensional fault height	$\beta = H_f/H_d$
adimensional upstream head	$\gamma = H_u/H_d$
adimensional upstream distance	$\sigma = D_u/H_d$
adimensional wall head loss	$\varepsilon = h/H_d$

Charts Of The First Type

The adimensional seepage flow q is plotted with regard to adimensional uspstream distance σ.
Adimensional upstream head γ and wall head loss ε are parameters of the two nets of

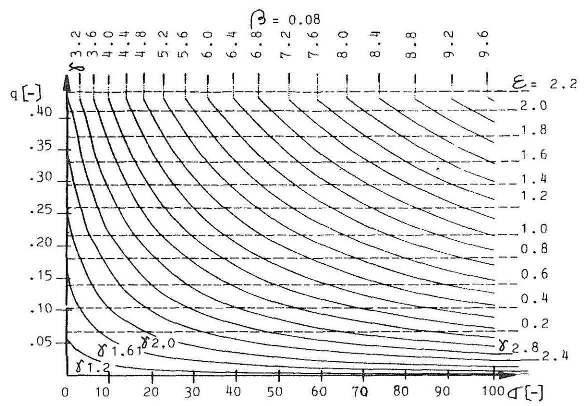


Fig. 7. Adimensional seepage flow q (Q/KH_d) versus adimensional upstream distance σ (D_u/H_d) (chart of first type)

curves. β is a constant for a given chart. An example of one of the charts of the first type is given in fig. 7 (for β = 0.08)

Charts Of The Second Type

The adimensional wall head loss ε is plotted with regard to the adimensional upstream head γ. Adimensional fault height β and seepage flow q are parameters of the two nets of curves. Adimensional downstream distance σ is a constant for a given chart. Fig. 8 gives an example of the charts of the second type. (for σ = 20)

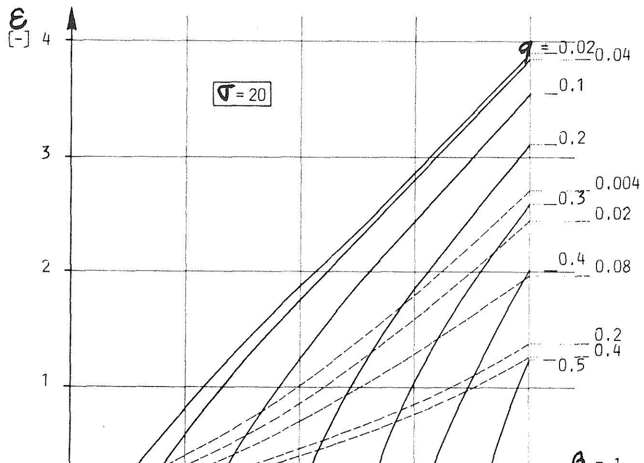


Fig. 8. Adimensional wall head loss ε (h/H_d) versus adimensional upstream head γ (H_u/Hd) (chart of second type)

Charts Of the Third type

The maximum seepage flow is obtained with the greatest height of fault, i.e. $H_f = H_d$.

The seepage ratio is defined as the ratio between the real seepage flow Q and the maximum seepage flow, all other geometric characteristics remaining the same.

This seepage ratio is plotted with regard to the adimensional fault height β. The adimensional upstream head γ is the parameter of the curve. The curves which reflect the drastic effect of small faults confirm the observations already made for cut-off walls below dams, however with different

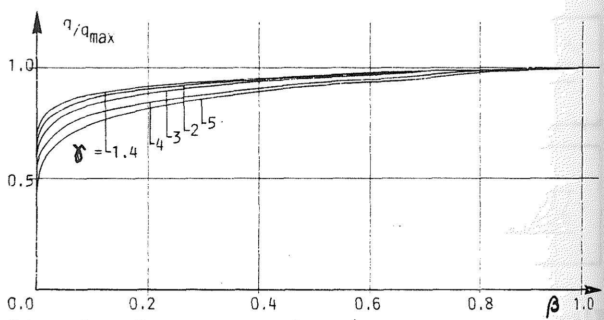


Fig. 9. Seepage ratio q/q max versus adimensional fault height β (chart of third type)

boundary conditions.

These charts of the third type are illustrated by a example given in fig. 9.

CONCLUSIONS

Analytical methods are not well suited to the study of the flow resulting from the existence of deep faults in cut-off walls.

The detailed parametric study was therefore carried out with numerical methods and more particularly with the Finite Element Method. The analysis of the obtained results outlines the existence of a non-seepage corner at the upstream contact with the wall. It also enabled us to work out approximate formulations which faithfully agree with numerical results.

The proposed charts allow one to determine the characteristics of the seepage (flow, wall head loss, height of the fault) from the geometrical parameters of the flow domain.

As an extension of the conclusions derived from the presented data, the influence of the thickness of the wall and of the location of the fault in the wall can be summarized here.

The influence of the thickness of the wall on the seepage flow is only significant for small heights of faults.

The location of the fault in the wall has no determining influence on the value of the seepage flow.

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