MODELLING OF DYNAMIC BEHAVIOUR
AT THE PILE BASE

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1. INTRODUCTION

The base resistance acting during pile driving is not yet very well understood. In the model proposed by E. Smith (1960) for wave equation analyses, this resistance is locally represented by a single degree of freedom system with the following three parameters : ultimate "static" resistance, quake and damping coefficient. Whereas the ultimate base resistance can be related to in-situ tests or other pile design concepts, the quake and in particular the damping coefficient are rather subjectively chosen. Correlations do exist between these two parameters and other soil properties but the scatter is very large.

In this paper an attempt is made to choose the relevant soil parameters on a rational basis, consisting of the following steps :

- examine the problem under static loading conditions to define a spring constant,
- examine the problem under harmonic loading,
- generalize the solution to the case of large strains.

The important advantage of this approach is that each chosen parameter has a physical meaning and can be determined experimentally or by other rational methods.

2. SEPARATION OF BASE AND SHAFT RESISTANCES

The analysis of the total pile resistance during driving is subdivided into the base resistance and the shaft resistance. It is therefore assumed that they can be examined independently, provided local forces and geometrical conditions are taken into account. The model which is extended here has been initially suggested by Randolph and Wroth (1978) for the static analysis of piles in elastic media. These authors visualize the behaviour of the medium by separating it into two parts (fig. 1) :

- a half-space, taking care of the base resistance,
- a layer of depth D, taking care of the friction.

The cylindrical pile is also defined by its radius, R, whereas the behaviour of the elastic medium is governed by its shear modulus G (or its Young's

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modulus E) and Poisson's ratio ν.

This paper focuses in particular on the pile base resistance, as the analysis of the shaft resistance has been presented elsewhere (Holeyman, 1984b).

3. LINEAR BEHAVIOUR OF THE BASE RESISTANCE

The dynamic behaviour of a circular footing resting on a homogeneous elastic half-space (cf fig. 2) has been investigated previously. In particular, Lysmer (1965) has found that a single degree of freedom system can adequately reproduce the harmonic behaviour of a rigid footing subjected to vertical loading. This so called "Lysmer analog" is governed by the following equation:

$$m \frac{\partial^2 u}{\partial t^2} + \frac{3.4 R^2}{1 - \nu} \sqrt{\rho G} \; \frac{\partial u}{\partial t} + \frac{4 GR}{1 - \nu} u = Q \qquad (1)$$

with : u : vertical displacement of the rigid footing
- t : time
- m : mass of the footing
- ρ : mass density of the medium
- Q : vertical force acting on the footing.

Lysmer's analog has brought a simplification in the understanding of the dynamic behaviour of a rigid footing. In equation (1) the static stiffness $k = \frac{4 GR}{1 - \nu}$ and the damping coefficient $C = \frac{3.4R^2}{1 - \nu} \sqrt{\rho G}$ are defined accordingly.

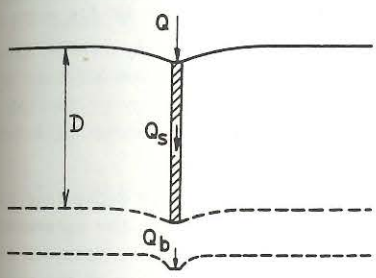

Figure 1 : Uncoupled model of shaft and sand base resistance.

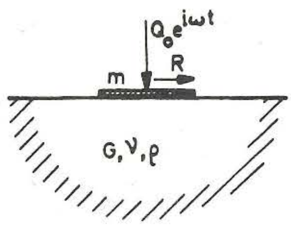

Figure 2 : Circular footing on top of elastic half-space.

The latter is the expression of the impedance of the half-space respective to the speed of the footing and is referred to as the "geometrical damping". It depends only on the mass and elasticity constants of the soil and on the geometry of the problem.

Due to its simplicity, Lysmer's analog cannot be readily extended to a non-homogeneous half-space, nor to a non-linear medium. In order to include essential factors, such as heterogeneity and non-linearity, it has been necessary to locally model the soil behaviour. The model presented here uses the concept of the equivalent solid : below the footing, the half-space is replaced by a solid of finite lateral extent (fig. 3) whose lower boundary has to obey certain conditions.

Only the vertical mode of deformation of the axisymmetric solid is considered and the parameters defining its shape and behaviour were determined by Holeyman (1985) taking into account the static behaviour and local impedance of the solid. The enforcement of these conditions requires that the equivalent solid is a truncated cone, having the same mass density as the soil medium and resting on a spring-dashpot (Lysmer's analog).

The modulus of elasticity of the solid in the vertical direction E_v is defined by :

$$E_v = \frac{G}{0.85\,(1-\nu)^2} \qquad (2)$$

whereas the radius r of the equivalent solid is defined as a function of the depth z by :

$$r(z) = R + \frac{1-\nu}{\sqrt{0.85}}\,z \qquad (3)$$

The vertical extent of the solid can be chosen at any value H, provided that the base rests on a single degree of freedom system defined by its spring constant k_H and its damping coefficient C_H :

$$k_H = \frac{4\,Gr_H}{1-\nu} \qquad (4) \qquad \text{and} \qquad C_H = \frac{3.4\,r_H^2}{1-\nu}\sqrt{\rho G} \qquad (5)$$

It is remarkable that the simple shape of this solid is solely determined by the value of Poisson's ratio ν. In spite of its simplicity, the agreement with Lysmer's analog is excellent in terms of the magnification factor M as functions of the non-dimensional frequency a_0 for various values of the mass parameter B_z, cf fig. 4.

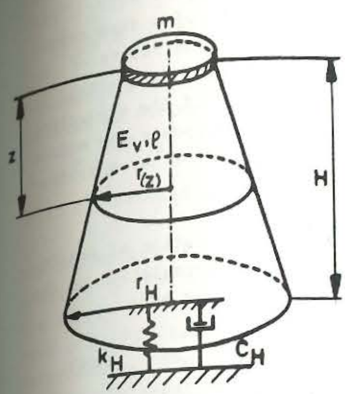

Figure 3 : Concept of the equivalent solid.

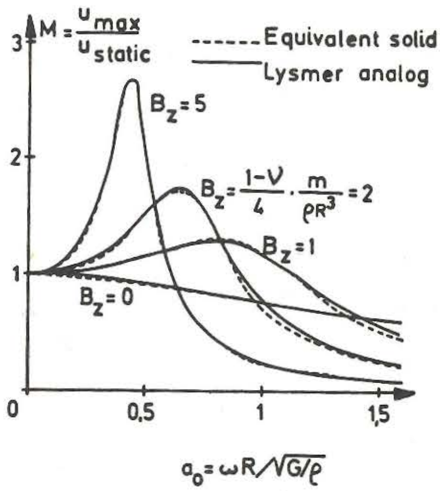

Figure 4 : Dynamic response curves of a rigid footing.

4. NON-LINEAR BEHAVIOUR OF THE BASE RESISTANCE

A first advantage of such a model is that it can be easily discretized into lumped parameters which are determined on the basis of local properties at each depth. Another advantage is that the intrinsic behaviour of the solid in the vertical direction can be more sophisticated than the linear elastic law. The hyperbolic law (Kondner, 1963) in particular has been used because of its simple mathematical formulation and because it can be defined by only two parameters : the ultimate strength and the initial modulus of deformation. It can be expressed in a non-dimensional form :

$$\frac{\varepsilon}{\varepsilon_o} = \frac{\eta}{1 - \eta} \qquad (6)$$

with

η : mobilization ratio defined by $\eta = q/q_r$, where (7)

q : vertical stress at the base

q_r : ultimate strength at the base

ε : strain

ε_o : reference strain (Hardin & Drnevich, 1972) defined as :

$$\varepsilon_o = \frac{q_r}{E_i}\text{, where } E_i \text{ : initial tangent modulus.} \qquad (8)$$

For load reversal, as occurring during pile driving, the unloading path is governed by the initial tangent modulus, thus assuming loss of energy. This hysteretic behaviour leads to the damping of the movement of a free system which is termed hysteretic damping. This damping due to the nature of the intrinsic behaviour law is not related to the rate of loading (velocity). It is solely dependent on the stress path. At the lower end of the equivalent solid, the stress level is so small that for practical purposes the medium below can be considered as linear elastic and can be replaced by a spring-dashpot.

Viscosity effects can also be incorporated similar to the notion of damping as proposed by Smith (1960). To respect the elementary formulation of the discretized model, the deformation behaviour can be expressed by :

$$\sigma = E\varepsilon + E' \frac{\partial \varepsilon}{\partial t} \tag{9}$$

with : $\frac{\partial \varepsilon}{\partial t}$ unit deformation velocity or strain rate

E' modulus of viscosity

With the viscoelastic model described by equation (9), the energy loss during a full loading cycle is dependent on the loading and unloading rate. The hysteresis loop is defined by this velocity difference and does not depend on the stress path as such. The damping associated with this behaviour is termed viscous damping, it is intrinsically generated.

Extensive experiments have been conducted concerning rate effects on soil strength. Starting from very low to very high rates of deformation, Holeyman (1984a) has surveyed the literature with respect to rheological studies, quasi-static analyses, constant rate of penetration tests, loading tests, pile driving tests, low velocity and high velocity projectile impacts etc. No general conclusions could be found : at best one can use the law which is the most appropriate to the velocity range at hand. These experiments yield the ultimate failure load obtained at some velocity of loading speed, which must not be confused with viscous damping as expressed by equation (9) and which can only be determined in a large strain test. Also the viscosity effect of water may play a role, though neither tests in dry sands lead to consistent conclusions. However, some experimental data indicate a strong non-linear effect expressed by a "resistance damping law" of the type : (Gibson and Coyle, 1968, Heerema ,1979, 1981, Litkouhi and Poskitt, 1980)

$$q_{r,d} = q_{r,s}(1 + J.v^N) \tag{10}$$

with : $q_{r,s}$: "static" strength at the base

v : velocity
J : "damping" coefficient
N : empirical exponent, usually close to 0.2

To avoid the sensitivity for very low loading rates, this relation has been modified as follows :

$$q_{r,d} = q_r^{ref} \quad (1 + v^{0.2})/1.46 \tag{11}$$

with q_r^{ref} defined as the strength computed at a reference velocity 0.02 m/s.

The instantaneous force in the soil elements below the base is computed by addition of the static resistance, as given by the hyperbolic law (eq. 6), the strain history and by the intrinsic viscous additional term, as given by equation (9), taking into account the viscosity modulus and the rate of deformation. The displacements are then computed by the explicit integration of the equations of motion and therefore take into account the effects of the geometrical damping, as in a traditional wave equation analysis. The instantaneous total force is then limited to the value given by the velocity dependent ultimate resistance cf. equation (11).

With the suggested model, combining all these features it is now possible to evaluate experimentally from a driving test the various parameters and to assess the relative importance of the different types of "damping" :

- Hysteretic damping, modelled by the hyperbolic law (eq. 6),
- viscous damping, modelled by the modulus of viscosity (eq. 9),
- geometrical damping, modelled by the geometry, the masses and the tangent modulus (equations 2 to 5),
- resistance damping, modelled by the non-linear velocity dependent law (eq. 11).

It can be noted that the parameters involved, along with the ultimate strength and the initial tangent modulus can be assessed by conventional laboratory or in-situ tests.

5. FIELD EXPERIMENTS

Steel piles driven for an experimental pile test programme in Belgium were used to analyze dynamic pile behaviour. The steel tubes with an outside diameter of 0.6 m and provided with a heavy bottom plate were driven into a dense, saturated tertiary sand layer, as shown by the static cone penetration test (CPT) in figure 5. Pile instrumentation was particularly extensive close to the base plate and consisted of accelerometers, strain gages and a velocity transducer. This velocity transducer was installed as suggested by Holeyman (1984a) and was found to give more reliable information than conventional accelerometers, as integration problems could thus be avoided.

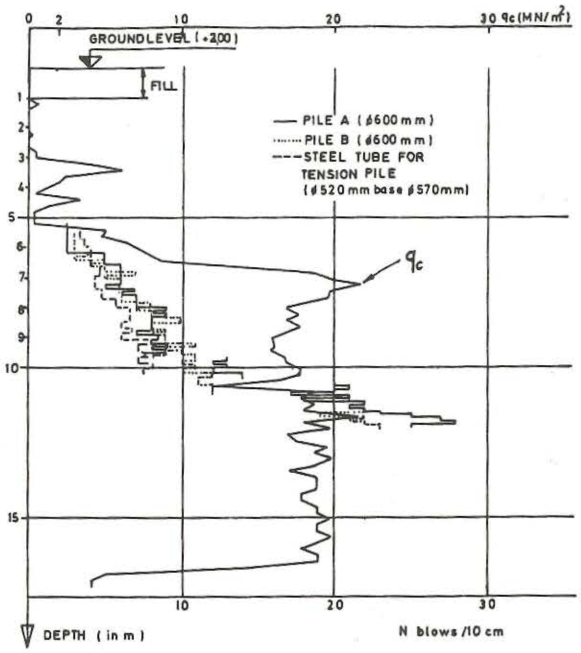

Figure 5 : Diagram of cone resistance (CPT) and pile driving resistance.

The signals recorded at the base during driving with a Diesel hammer (Hera 5700) into the dense sand are shown in fig. 6 : force and velocity times impedance Z of the pile as functions of time.

The parameters of the model enabling the reproduction of the recorded phenomena were determined by an iterative trial and error method, using one of the measurements (velocity or force) as an imposed condition, and comparing the measured values (force or velocity) with the computed one, using the modelled equivalent solid concept. These analyses were conducted using a wave-equation code enabling computation of displacements at various levels in the pile and in the soil below the base, taking into account the inertia effects of the heavy bottom plate.

To reduce the numberical efforts for sands, a value of Poisson's ratio equal to 1/3 was chosen. It became obvious that the value of the modulus of viscosity had to be chosen very small (0.02 MPa/s.). Besides the resistance damping, the two major parameters defining the resistance at the base were the reference ultimate base resistance : q_r, and the reference strain ϵ defined by equation (8).

These two important parameters were defined first by introducing $J = 0$ in formula (9). A number of more or less equivalent solutions could be found. In particular, for $q_r = 12$ MPa and $\varepsilon_o = 0.032$, a very good match was obtained between the computed and the measured force diagrams cf. fig. 6. For the maximum displacement (12 mm), the mobilized unit base resistance was 8.1 MPa. As this maximum displacement occurs at zero velocity, other mobilization curves passing through this experimental point would be able to give other suitable matches of the curves. This was the best fit found by trial and error. Three of the corresponding mobilization curves of the static base resistance of this base are shown in fig. 7 as a function of the base displacement. They pass through the same point around $q = 8.1$ MPa for $s = 12$ mm. They are characterized by the following parameters : $q_r = 10$ MPa with $\varepsilon_o = 0.023$, $q_r = 12$ MPa with $\varepsilon_o = 0.032$ and $q_r = 14$ MPa with $\varepsilon_o = 0.040$.

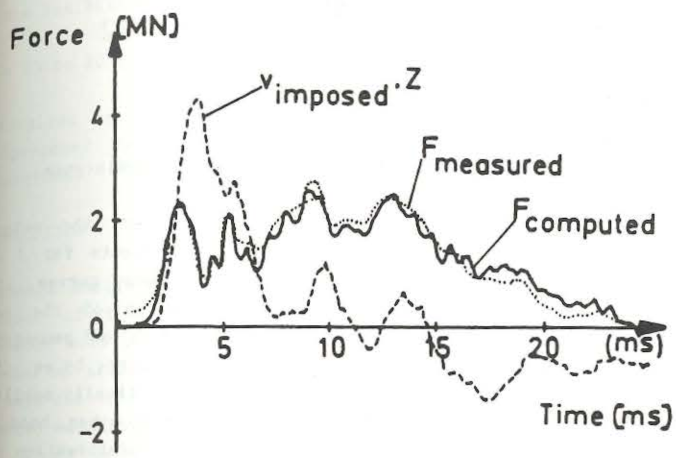

Figure 6 : Measured and matched computed force diagrams from field test data in sand.

For the mobilized stress range , these curves are very similar but beyond, they yield different ultimate values. The deduction of the ultimate bearing capacity is thus a matter of extrapolation, just as is the case in an incomplete static pile load test, i.e. one has to estimate the ultimate failure load.

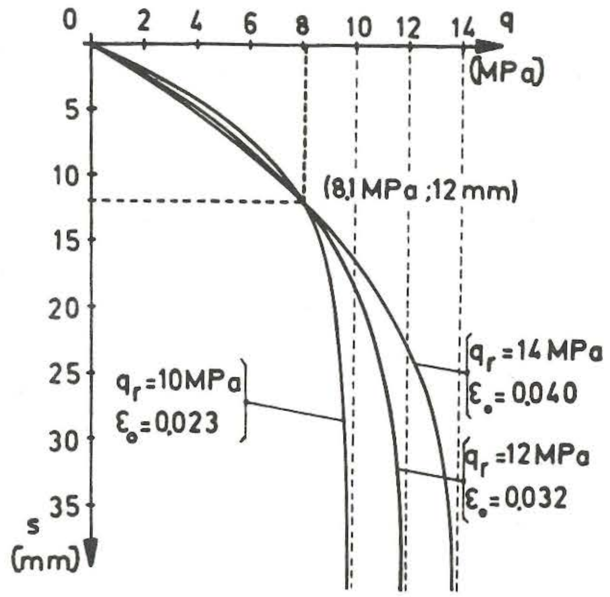

Figure 7 : Mobilization curves of the static base resistance.

The last factor to be addressed is the significance of the velocity dependency of the ultimate resistance. Various coefficients for J were tried, without significant effect on the calculated response curves. This can be explained by the fact that during mobilization of the total compression force in any soil element, the intrinsic viscous and geometrical dampings are not sufficiently mobilized for the upper bound set by eq. (11). Indeed, on the one hand when the velocity is high, the statically mobilized force is far from its ultimate value, whereas on the other hand the statically mobilized force is close to failure while the deformation rate is very small and decreases to zero. For this effect to be felt, the pile needs to undergo large sets at high loading rates, which is only the case for very weak soils or for piles with a small base. Thus under normal driving conditions and specially at the end of driving where the transient maximum displacements at the base are small, the velocity dependency of the ultimate resistance should not be of significance. However, the significance of velocity dependency of the modulus of deformation needs to be further investigated.

When comparing the various matching curves, the one corresponding to q_r = 12 MPa and ε_o = 0.032 was selected as the best estimate. The ultimate static resistance during driving is somewhat smaller than the one computed according to De Beer's method (1971-1972) : 14.3 MPa. Further analyses concerning the skin friction (Holeyman (1984b) using dynamic pile test data measured at the top of the pile suggest very high values (τ_f = 0.2 MPa) towards the lower end of the pile shaft. This seems to give satisfactory agreement on the total resistance, and one can consider that the dynamic base resistance is the one acting at the very base and probably does not take into account the stress level induced by the shaft friction corresponding to the static solution. These deductions were used to compute the pile deflection curve under static loading. The comparison of this curve with the one obtained from a static loading test carried to failure suggests that the static friction after driving was about 50 % higher than the one determined from the driving test. This value was confirmed by a pull out test which reinforces the validity of the assumptions made for the base resistance.

Similar deductions were made for other piles and in particular for closed end Franki steel tubes driven into dense sand (Holeyman, 1984a), leading to the following empirical relationship between the initial tangent modulus and the cone resistance q_c obtained from CPT tests :

$$E_i = 15 \text{ to } 20\ q_c \qquad \text{for } 10<q_c<30 \text{ MPa} \qquad (12)$$

These values tend to be slightly in excess (30 %) of those obtained from data proposed by Hardin and Drnevich (1972) for earthquake engineering applications.

6. CONCLUSION

A rational analysis procedure is proposed modelling the dynamic non-linear behaviour of the base resistance of piles during driving. Based on measurable soil data and in particular on the results of CPT tests, it is possible to assess the static load-deformation characteristics below the pile base and an additional resistance term attributed to loading rate. The suggested model which replaces the half-space below the base by a truncated cone allows separation of geometrical damping, hysteretic damping and viscous intrinsic damping and resistance damping. Experimental data suggest that under normal driving conditions only the geometrical and the hysteretic damping are of significance. From a pile driving test, the loading curve up to the maximum transient displacement can be determined reliably, while the ultimate bearing capacity can only be estimated by extrapolation.

7. REFERENCES

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8. SUMMARY

A physical one-dimensional model is presented to simulate the dynamic behaviour of the soil below the base of a pile during driving. In this simplified approach, the soil medium is replaced by a truncated cone resting on a single degree of freedom system. After checking the agreement between the discrete model and the homogeneous elastic medium, the behaviour of a non-linear medium is investigated. The resulting discrete model can be readily incorporated into a wave equation model, since only additional soil elements have to be added below the base of the pile.

The advantage of such a model is the possibility to separate the geometrical damping from the viscous and hysteretic damping. It is demonstrated that the use of such a model reproduces accurately the behaviour of a pile base, as confirmed by field data using transducers located at the toe of a closed-end steel pile, driven into a dense sand. The model has also been successfully used in more classic analyses of force and velocity measured at the head of piles.