COMPARISON OF TWO MODELS TO EVALUATE THE BEHAVIOUR OF A VIBRATORY DRIVEN SHEET-PILE.

By

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Abstract.

A method to predict the average penetration speed of vibratory driven sheet piles is presented. In a first part, a simple model integrates the equation of movement where the sheet pile is modelled as a rigid body and is loaded with a static weight combined with a cyclic load. In this model, the soil is supposed to have perfect plastic behaviour. The skin friction is supposed to be reversible and acting in a direction opposite to the movement of the sheet-pile, while the resistance at the toe level is a step function whose value is zero when the sheet pile moves up and is constant when the sheet pile moves down.

In a second part, the model of the soil behaviour is extended to an elasto-plastic model. The authors describe the difficulties of the numerical integration and compare the results provided by the two models.

1. Introduction

Three basic driving techniques can be used to install piles or sheet-piles in the soil: impact driving, vibratory driving and jacking. The most commonly used method in soft soils is vibratory driving because it allows a fast installation without environmental disturbances. Impact driving can generate high energy under difficult soil conditions, but generates noise and high vibrations in the soil. Therefore, impact driving cannot be used near other constructions. Jacking is expensive and is used only where sensitive environmental conditions are encountered.

Vibratory driving is often used to install steel sheet-piles. A vibrator induces to the soil a cyclic load. The first effect of this load is to apply a strength greater than the static resistance of soil. The second and the more important effect is to lead to a significant reduction of soil resistance. Indeed, the result of the cyclic movement is the degradation of soil strength and the build-up of the pore pressure. The ultimate build-up of pore pressure in saturated sands is referred to as liquefaction, expressing a complete loss of strength. Some consider that the pile is not actually installed into the ground primarily by the vibrating force but rather by sinking the pile into degraded material under gravity forces.

A model to predict the drivability speed during vibratory driving is an essential tool for the contractors. However, creating this model is a difficult operation because many parameters influence the resistance of soil. Therefore, we follow a step by step method from the more easy situation to the more realistic. This way, we can understand the importance of each element. The first part of this paper presents the results of the preliminary developments: we simplify the behaviour of soil in a perfectly plastic model without degradation. In the second part, we introduce an elasto-plastic model to describe the soil resistance with a more realistic geometric model. This second part will show the importance in the choice of the shear modulus.

2. Plastic model

This model aims to calculate the penetration speed of a sheet-pile at a given depth in function of the soil resistance. The behaviour of the soil is assumed to be plastic and without degradation.

2.1. Assumptions.

In this simple model, the following assumptions are considered:

- The sheet-pile is considered as a rigid mass.
- The strain-stress law of the soil is perfectly plastic. The shaft resistance is directly mobilised in the opposite direction of the movement and the base resistance is only mobilised when the sheet pile moves down.
- The water table is deep enough not to influence the penetration speed of the sheet pile.
- We do not consider the degradation induced in the soil resistance by the cyclic loading.

In summary, the behaviour of the sheet-pile in the soil is like a rigid block that slips on a surface, as described by Newmark (1965).

2.2. Analytical equation of the movement.

The forces applied on the sheet-pile during the penetration (figure 1) can be separated into (1) the mechanical action loads that causes the movement (weight F_w and the cyclic load F_c) and (2) the soil resistance that try to stop the movement (shaft resistance F_{shaft} and base resistance F_{base}). These strengths can be combined in the movement equation:

$$M_v.a(t) = F_w + F_c + F_{shaft} + F_{base} \tag{1}$$

where $M_v.a(t)$ is the inertial strength of the sheet-pile, a(t) the acceleration and M_v the vibrating mass.

The first action load is the static weight (F_w) of the sheet-pile and the vibrator. It is calculated with the relation:

$$F_w = (M_s + M_v).g \tag{2}$$

where M_s is the static mass of the vibrator and M_v the vibrating mass of the vibrator plus the mass of the sheet-pile.

The second action load is the cyclic load (Fc) applied by the vibrator upon the soil. This vibrator is formed with two eccentric masses that turn in opposite directions and create a cyclic strength calculated by :

$$F_c = m_e.\omega^2.\sin(\omega.t) \tag{3}$$

where m_e is the eccentric moment of the vibrator [kg.m], ω the circular frequency of the vibrator [rad/s] and t the time.

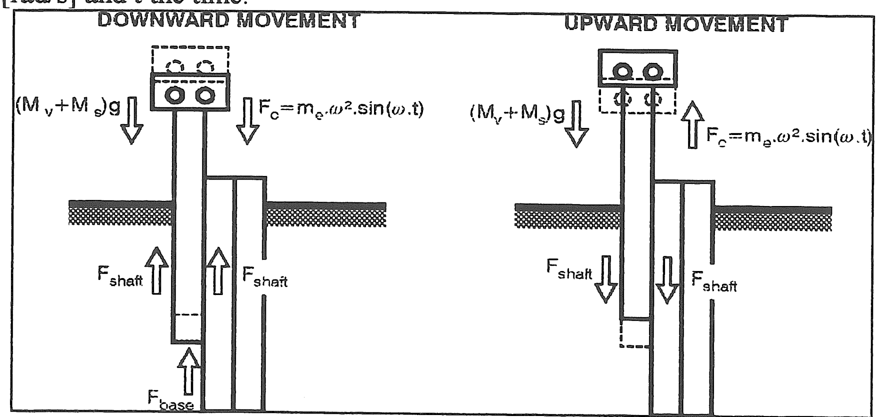

Figure 1: The different strengths applied to the sheet-pile.

There are two resistant strengths: the shaft resistance and the base resistance. These resistance try constantly to equilibrate the action loads but are limited by a maximum value $F_{s\,max}$ et $F_{t\,max}$. The big difference between these two resistance is the reversibility (figure 1). Indeed, the shaft resistance is always acting in the opposite direction of the movement, regardless of the direction. On the other hand, the base resistance is different from zero only when the sheet-pile moves down. The model considers these assumptions by calculating the maximum shaft resistance in function of the speed direction. Mathematically, The maximum shaft resistance can be calculated by

$$F_{shaft\,max} = \tau.\Psi.h.\,\mathrm{sign}(v) \qquad \text{with } \mathrm{sign}(v) = \begin{cases} 1 \text{ if } v > 0 & \text{(down movement)} \\ 0 \text{ if } v = 0 & \\ -1 \text{ if } v < 0 & \text{(up movement)} \end{cases} \quad (4)$$

where τ is the maximum shear stress, Ψ the perimeter, h the depth where the penetration speed is evaluated and v the sheet-pile speed.

By contrast with the shaft resistance, the base resistance is a step function where the value is zero when the sheet-pile moves up and positive only when the speed is bigger than zero. Mathematically, that can be described by

$$F_{t\,max} = q_b.\Omega.\left(1_+(v)\right) \qquad (5)$$

where q_b is the maximum compression stress of the soil, Ω the cross-sectional area of the sheet-pile and $1_+(v)$ is the step function (= 1 if v>0 and =0 if not).

Figure 2 summarises the assumptions regarding the different strengths adopted in this simple model. The figure shows the curve of the action load (static weight plus cyclic load) and the maximum resistance to the down and up movements. Three zones can be isolated: the zone A, B and C. In zone A and C, the action load is greater that the total soil resistance. Consequently, the sheet-pile is in unstable position and has a tendency to accelerate. Inversely, in the zone B the strengths are able to equilibrate the action load. The sheet-pile tries to reduce to zero the speed caused by the acceleration in area A or C.

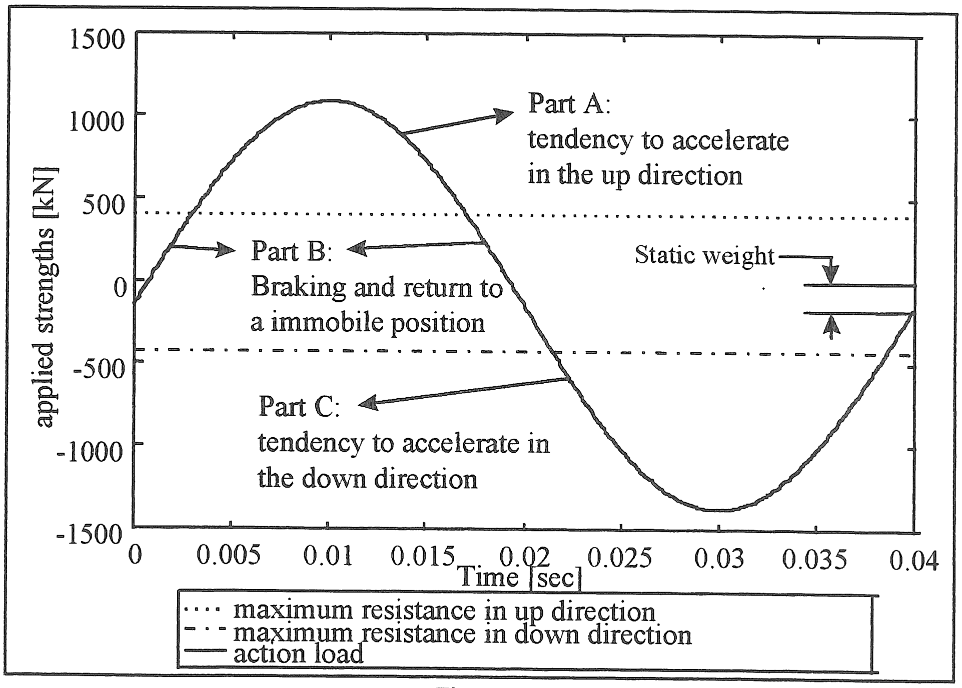

Figure 2

Considering these assumptions, the movement equation (equation 1) can be developed in the following differential equation:

$$M_v . a(t) = F_w + F_c + F_{shaft} + F_{base}$$

$$= \begin{cases} 0 & \text{if sheet - pile immovable and action load} < \text{resitance strength} \\ \underbrace{\left(M_s + M_v\right).g}_{\text{Weight}} + \underbrace{m_e.\omega^2.\sin\left(\omega.t\right)}_{\text{cyclic load}} - \underbrace{\tau.\Psi.\text{sign}\left(\dot{u}(t)\right)}_{\text{shaft resistance}} - \underbrace{q_b.\Omega.\left(1_+\left(\dot{u}(t)\right)\right)}_{\text{base resistance}} & \text{if not} \end{cases}$$

(6)

where $u(t)$ is the movement of the sheet-pile, $\dot{u}(t)$ the speed (= v) and $\ddot{u}(t)$ the acceleration (=a).

2.3. Numerical integration.

Analytical integration of equation 6 is a difficult operation. Therefore, we have adopted a numerical approach. We integrate the equation time-step by time-step during one cycle. At each time increment, the acceleration is evaluated by the equation 6 considering the speed calculated at the previous time increment.

To explain the method used, let's consider that at the time t, the acceleration a_t, the speed v_t, and the displacement d_t of the sheet-pile are known. To calculate these variables at the next increment of time $t+\Delta t$, the first step is to analyse the figure 2 to know which zone is met at the time $t+\Delta t$. Then, based on the speed v_t at the time t, the function sign(v) and $1_+(v)$ are evaluated. The acceleration at the time $t+\Delta t$ is calculated with the equation 6. Consequently, we can obtain the speed and the displacement with two integration of this acceleration. All the steps explained here are illustrated on figure 3.

To start this calculation, the first speed must be known. This is the main difficulty of the numerical resolution of the problem. An iterative approach is used to solve this issue. We integrate the acceleration on one cycle with a first selected speed. After the calculation, the speed at the end of the cycle is compared with the speed chosen initially. If the steady state is reached, the two values must be the identical. If it is the case, the calculation is finished. If not, a new iteration is realised with a new choice of the initial speed.

When the equality between the two speed is reached, the average penetration speed is calculated by the displacement of one cycle divided by the period of the cycle.

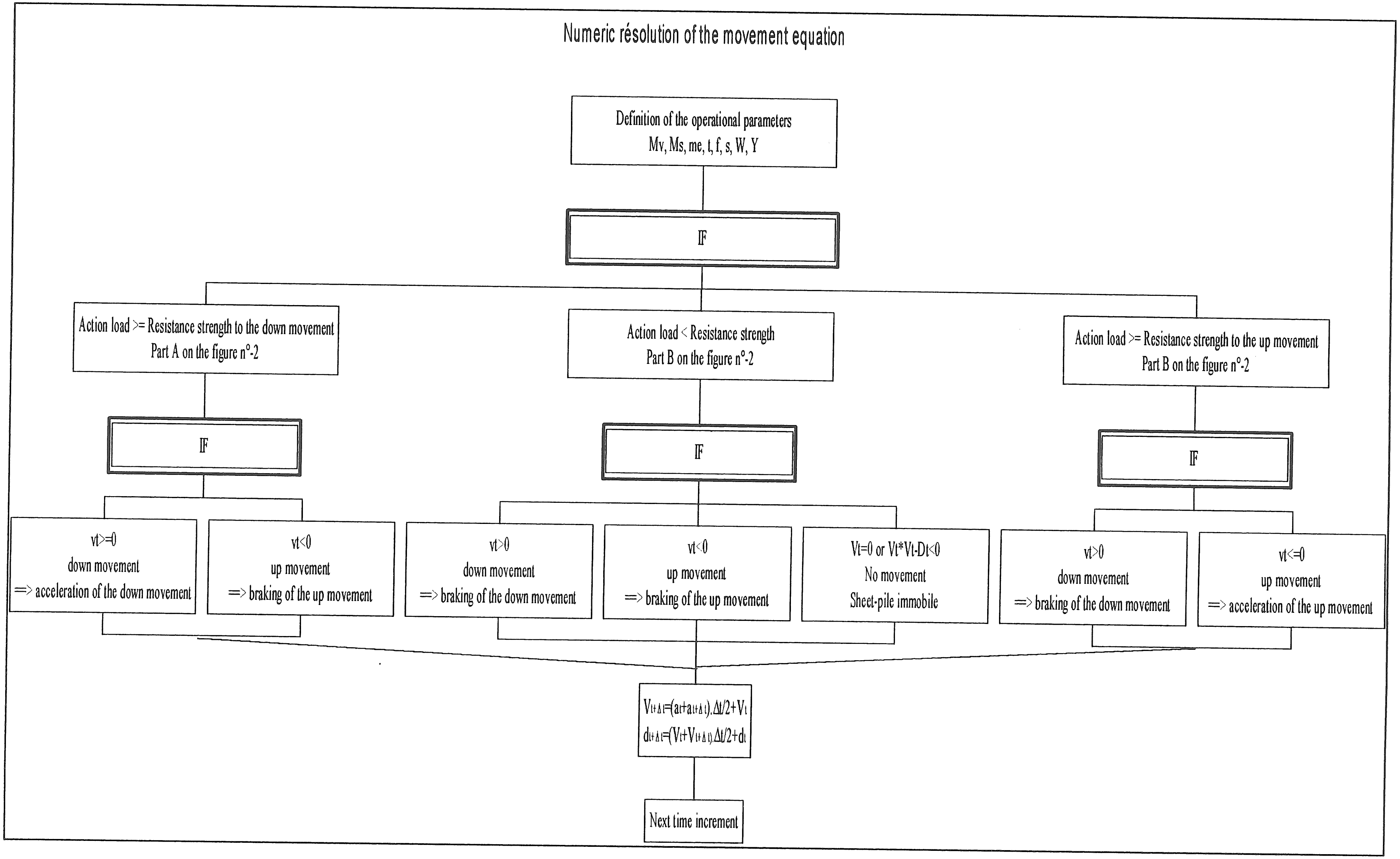
Numeric résolution of the movement equation
Definition of the operational parameters
Mv, Ms, me, t, f, s, W, Y
IF
Action load >= Resistance strength to the down movement
Part A on the figure n°-2
Action load < Resistance strength
Part B on the figure n°-2
Action load >= Resistance strength to the up movement
Part B on the figure n°-2
IF
IF
IF
vt>=0
down movement
=> acceleration of the down movement
vt<0
up movement
=> braking of the up movement
vt>0
down movement
=> braking of the down movement
vt<0
up movement
=> braking of the up movement
Vt=0 or Vt*Vt-Dt<0
No movement
Sheet-pile immobile
vt>0
down movement
=> braking of the down movement
vt<=0
up movement
=> acceleration of the up movement
$V_{t+\Delta t}=(a_t+a_{t+\Delta t}).\Delta t/2+V_t$
$d_{t+\Delta t}=(V_t+V_{t+\Delta t}).\Delta t/2+d_t$
Next time increment

Figure 3

2.4. Examples

In this paragraph, two examples for two different frequencies are discussed. The first one, with the lower frequency allows to understand the adopted assumptions. The second with a usual frequency shows the particularities induced by the choice of the assumptions.

The operational parameters used in these simulations are mentioned in the table 1.

Vibrating mass (M_v) = 10 000 kg	maximum base resistance (q_b) = 0 kPa
Static mass (M_s) = 5000kg	sheet-pile perimeter (ψ)= 2 m
eccentric moment (m_e) = 50 kg m	sheet-pile section (Ω)= 250 cm²
maximum shaft stress (τ) = 100 kPa	depth (h)= 2 m

table 1

The results of the first simulation with a frequency of 15 Hz is drawn on figure 4. The average penetration speed at this frequency is 9 m/min.

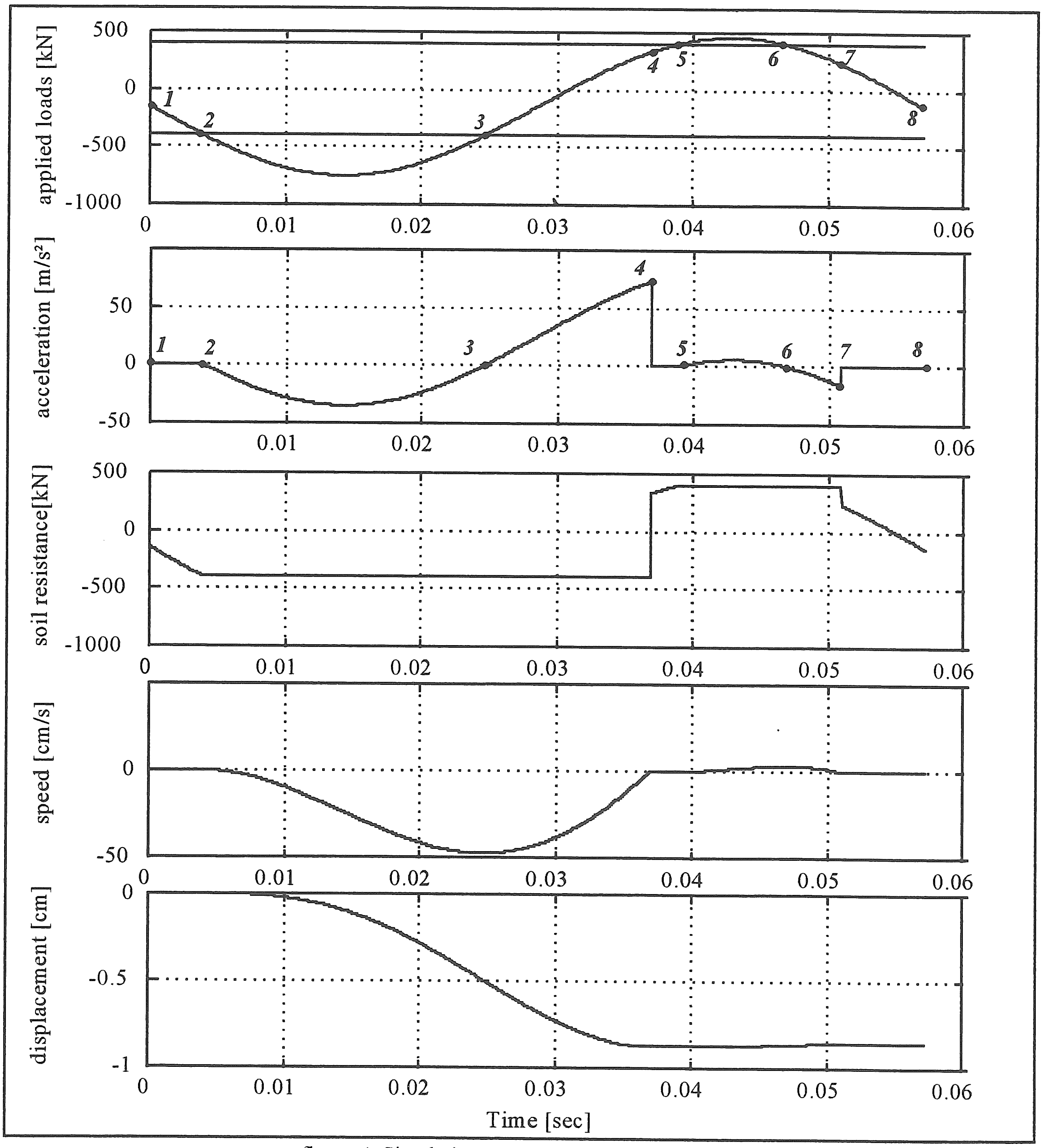

figure 4: Simulation results (Frequency = 15 Hz)

On figure 4, the different steps of the integration appear. Starting from time 1, at the first iteration, the sheet-pile is supposed immovable. Therefore, the initial speed equals zero. Between time 1 and 2, the action load and the resistance strength are equal and the sheet-pile stays immovable. But after time 2, an instability appears and an acceleration in the down direction is created. From time 2 to 3, the acceleration creates a speed and a down movement. At time 3 where the speed is maximum, the resistance loads are able to equilibrate the action load. But the speed at this time creates an inertial strength that is not equilibrated. The sheet-pile decelerates until the energy stored between time 2 and 3 is dissipated. At time 4 where the speed returns to zero, all the strengths are equilibrated and the sheet-pile stops. There is no movement until time 5, where a new instability appears in the up direction. From the time 5 to 7, the acceleration, the speed and the movement follow the same way in the up direction. At the time 8, one cycle after the time 1, the speed equals zero, the steady state is reached and the calculation is finished.

Figure 5 presents the results for a simulation with a frequency of 25 Hz. In this case, the sheet-pile does not stop during the cycle of load. For that frequency, the average penetration speed equals 22 m/min. The particularity of the acceleration is the jump on each change of direction of the movement. This result is induced by the choice of the perfect plastic model.

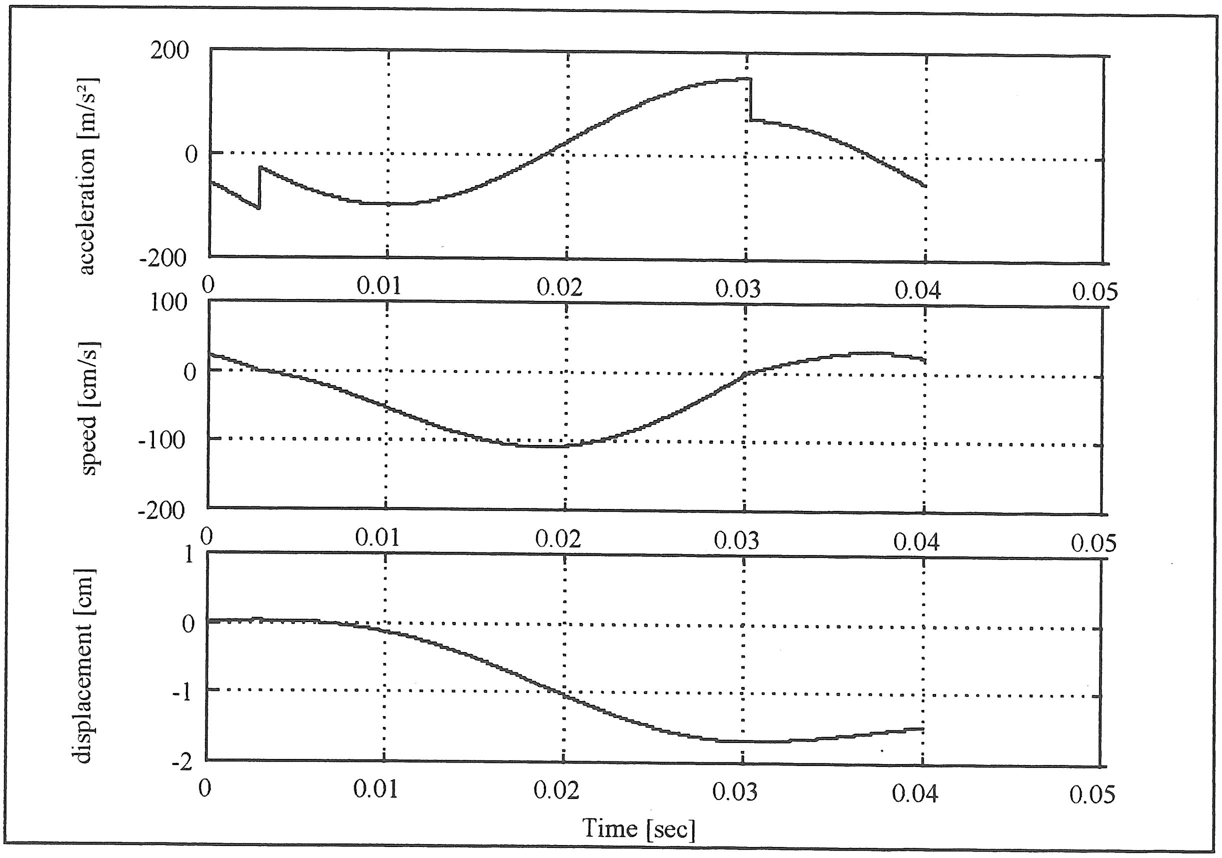

Figure 5: simulation results (frequency = 25 Hz)

2.5. Results.

On figure 6, the average penetration speed in function of the frequency of the vibrator for different shaft resistance is drawn. This graph show two interesting observations. The first is the existence of critical frequency f_c below which there is no penetration. This frequency is given by:

$$f_c = \frac{1}{2.\pi} \cdot \sqrt{\frac{\tau.\mathrm{peri.h} + \sigma.\mathrm{suf} - (M_s + M_v).g}{m_e}}$$

This equation was obtained by identifying the maximum action load and the maximum resistance strength. The second information is the quasi linear relationship between the average speed and the frequency. This observation confirms the feasibility of an equation that, based on operational parameters and some empirical values, would allow to calculate simply an approximate penetration speed, as suggested by Holeyman (1993a).

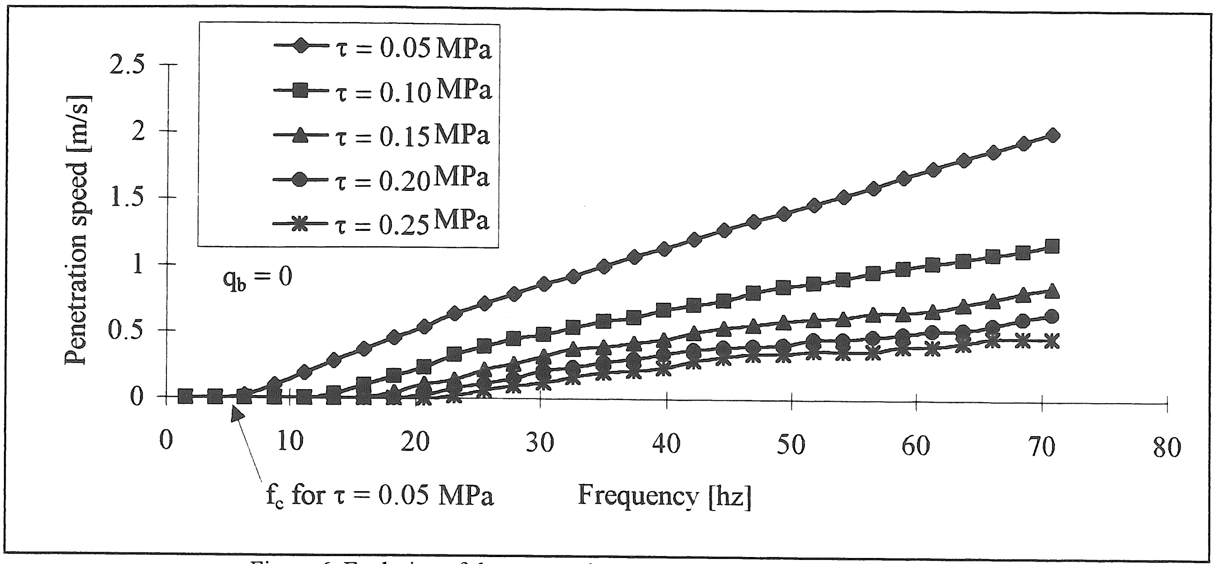

Figure 6: Evolution of the penetration speed in function of the frequency.

3. Extension to an elasto-plastic model.

3.1. Introduction.

On the way to the more realistic model, the plastic model is modified in an elasto-plastic model with a more realistic geometric model. The following paragraph describes the modifications in the assumptions presented in the paragraph 2.1.

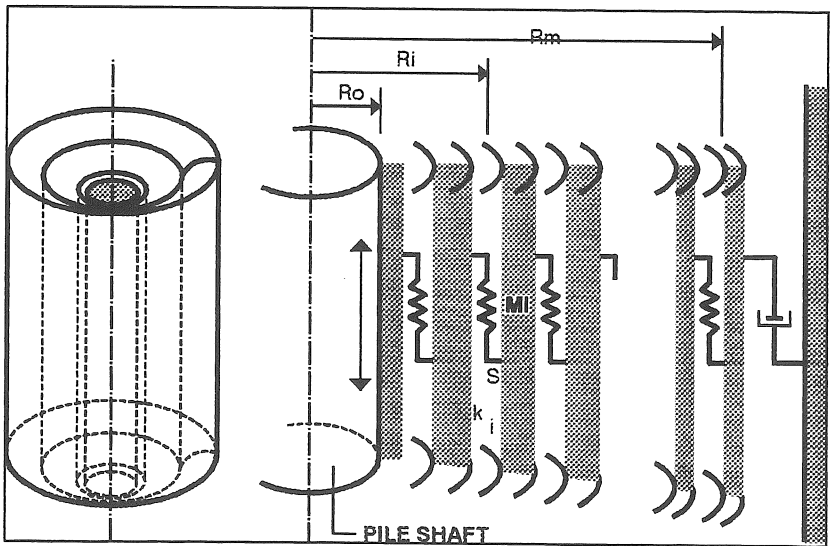

Figure 7: Geometric model.

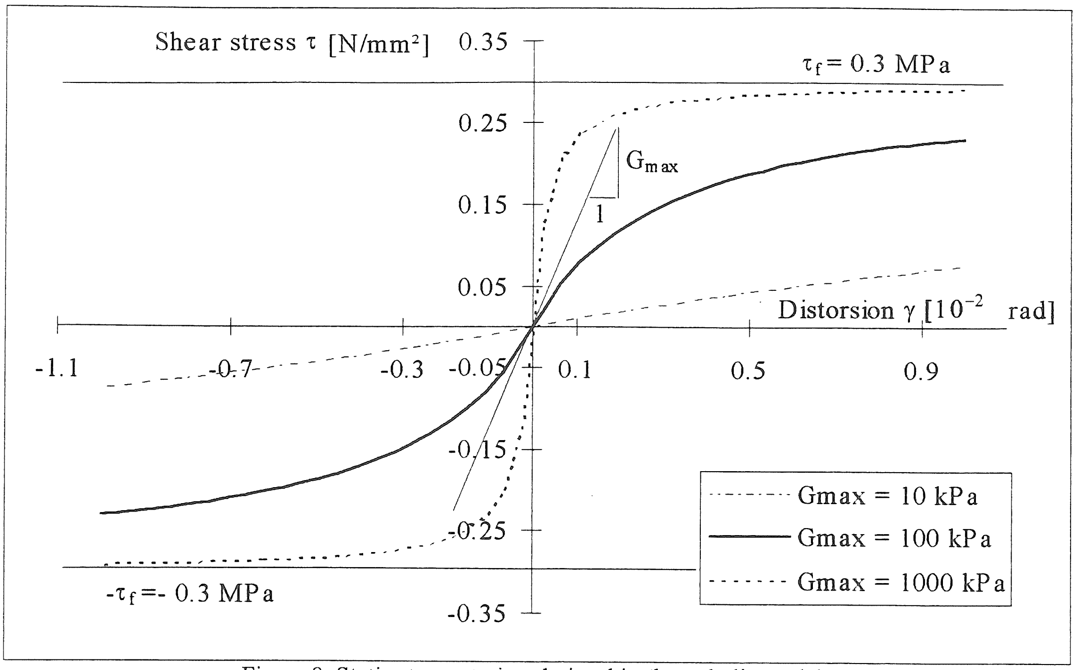

Figure 8: Static stress-strain relationship (hyperbolic model)

In the paragraph 2.1., we assumed that the sheet-pile and the soil were two rigid bodies that slip on each other. Now, we represent the soil by discretizing the medium into concentric rings that have their own individual mass (figure 7) and transmit forces to their neighbours (Holeyman ,1993b). The shear force-displacement relationship between successive rings is calculated based on an elasto-plastic stress-strain relationship. To evaluate this relationship, the model uses the hyperbolic law (Kondner, 1963) to describe the static behaviour of soil (figure 8) and the two Masing's laws (Masing, 1926) to represent the hysteresis observed during a cyclic loading (figure 9). That model needs two parameters: the maximum static resistance of soil τ_f and the initial (or tangent) shear modulus G_{max} (figure 8). On figure 9, the stress-strain relationship used is drawn for different initial shear moduli. This graph shows the importance of the choice of the shear modulus. The higher this modulus, the closer perfection is the plastic behaviour of soil.

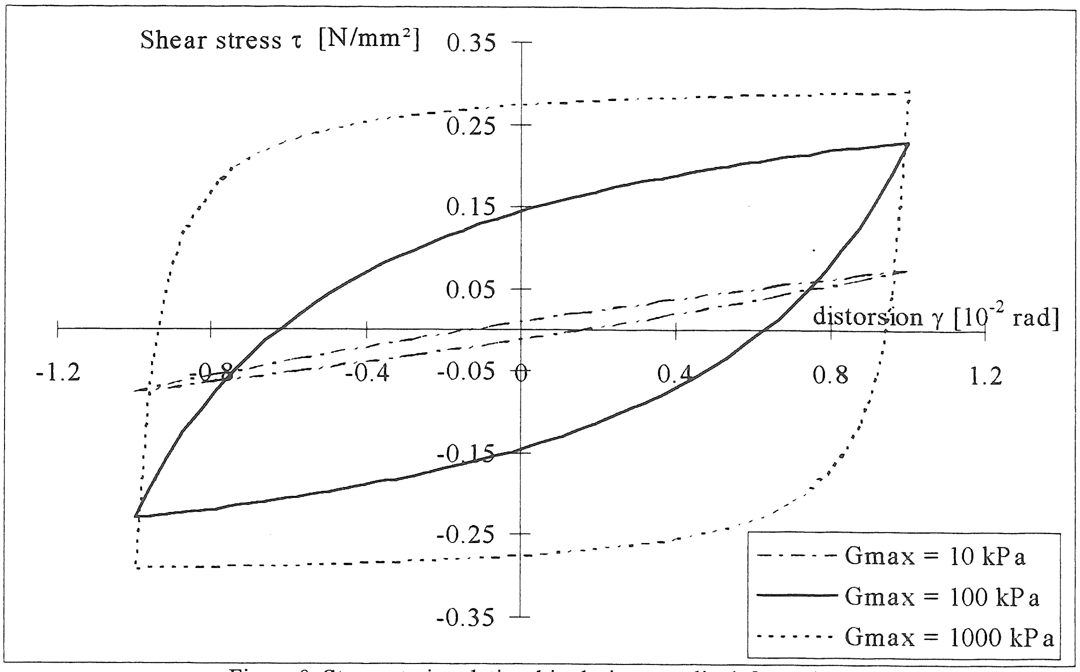

Figure 9: Stress-strain relationship during a cyclic deformation.

An energy absorbing boundary condition in accordance with plane-strain elasticity theory limits the lateral extent of the model at a distance large enough to ensure that deformations stay within the elastic range and to avoid artificial energy reflections.

The movement of the sheet-pile and the rings is calculated from the time integration of the law of motion with the same method explained for the plastic model: double integration of the acceleration resulting from the net unbalanced loads acting on each element.

3.2. Results and comparisons.

Figures 10 and 11 represent the result for two different values of the shear modulus G_{max}. In these simulations, the same values as in the example problem provided in paragraph 2.4 are used. The number of rings of the geometric model is 20 and the influence radius R_m is equal 2 meters.

For the shear modulus of 1000 MPa, the behaviour of the soil is almost perfectly plastic. The calculated average penetration speed (22 m/min) is nearly the same as the speed calculated with the plastic model (23m/min). With the shear modulus of 50 MPa (a more realistic value), the speed is lower and equals 17 m/min. On figure 11, the jumps in the acceleration curve observed with the plastic model (figure 5) disappear. Indeed, the effect of the elasto-plastic model is to smooth the acceleration curve by a non instantaneous inversion of the shaft resistance.

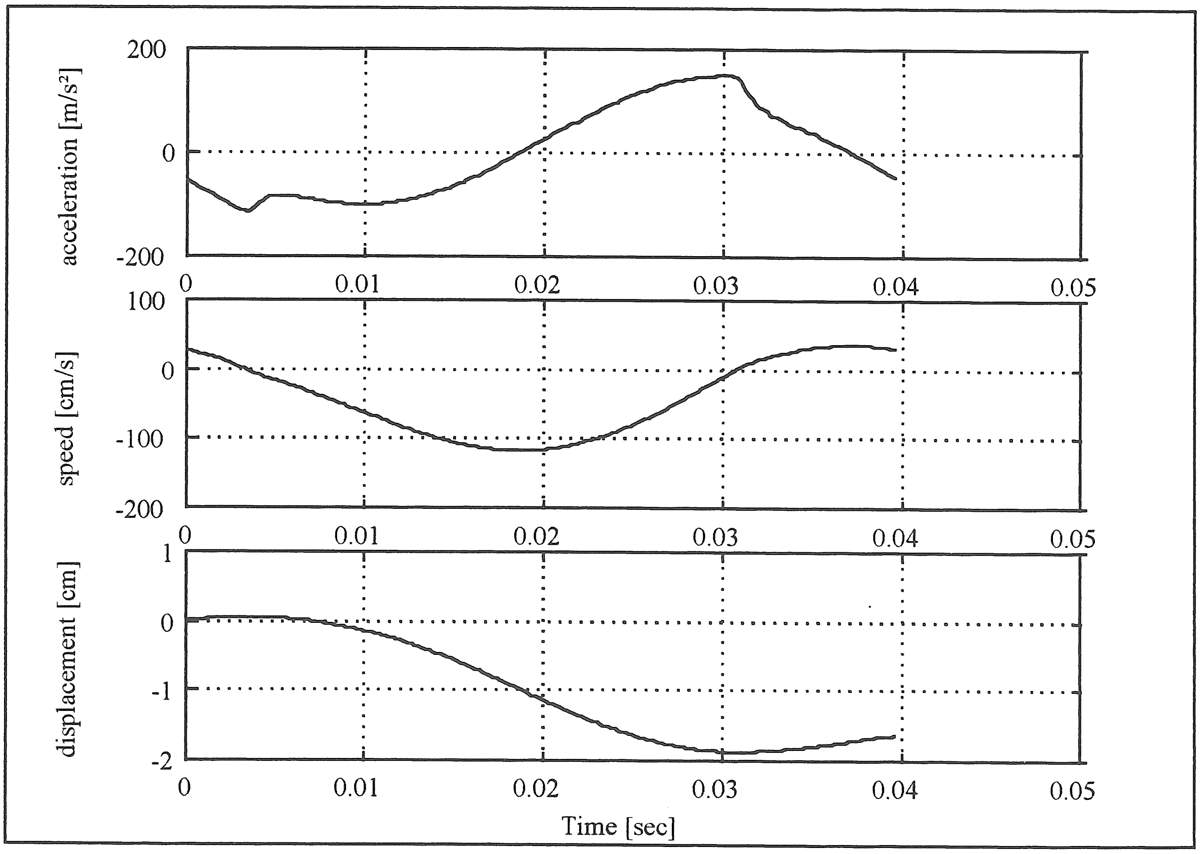

Figure 10: simulation results (G_{max} = 1000 MPa)

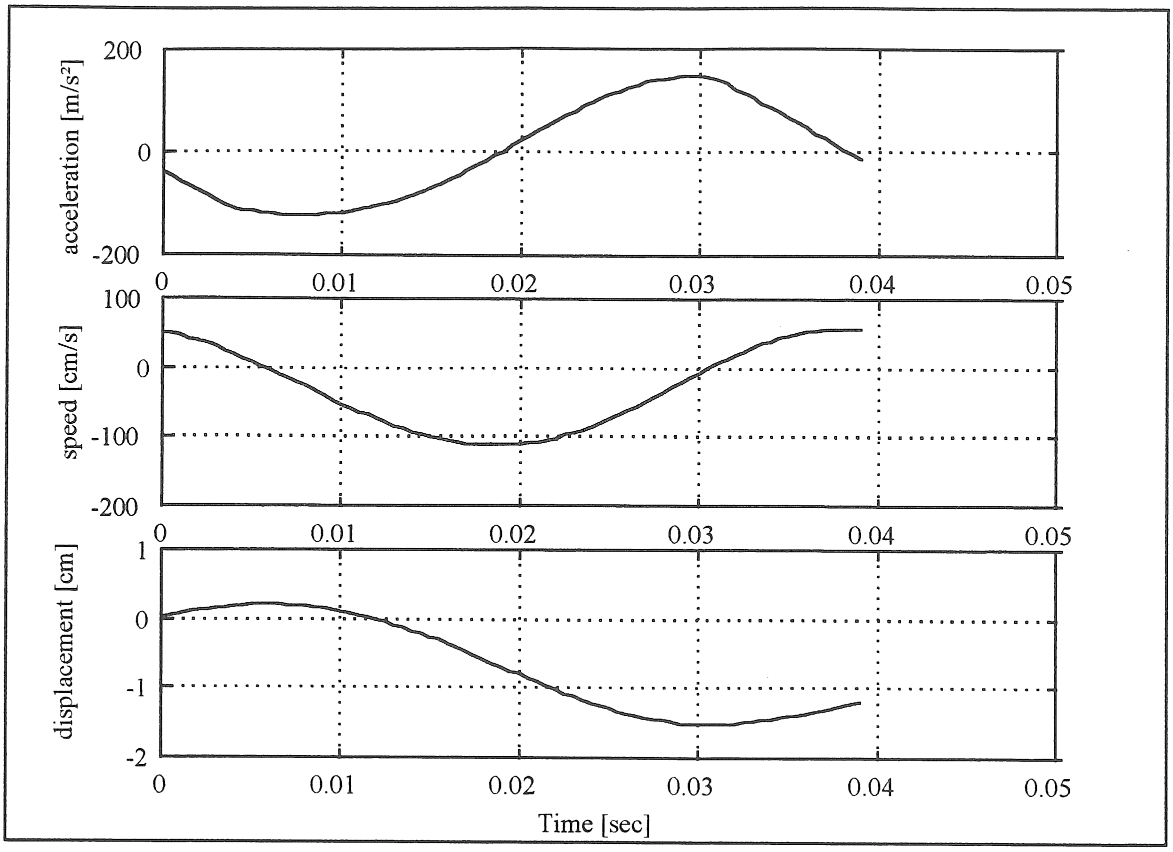

Figure 11: simulation results (G_{max} = 50 MPa)

These two examples show the importance of the choice of the initial shear modulus. To illustrate this observation, the evolution of the average penetration speed in function of the shear modulus is drawn on figure 12.

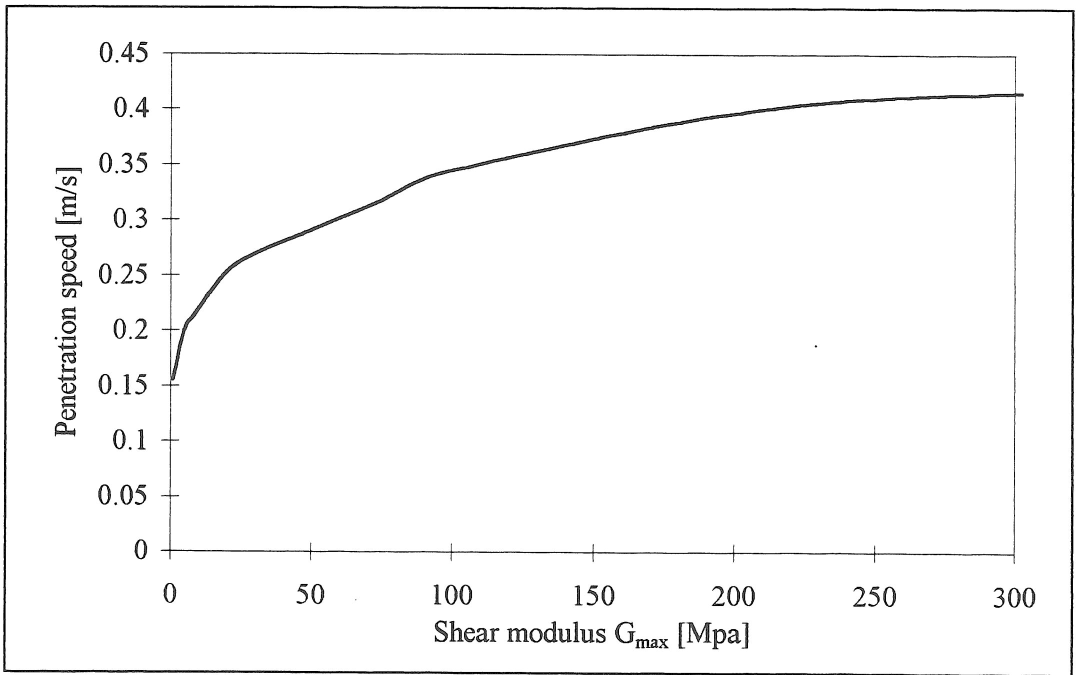

Figure 12: Evolution of the penetration speed in function of the shear modulus.

A sheet-pile penetrates easier in a hard soil than in a soft soil. This observation is explained by the fact that the sheet-pile moves up easier in the soft soil. This is shown clearly on figure 13. For the more perfectly plastic soil, the shaft resistance is directly mobilised and is directly opposed to the movement. On the contrary, the inversion of the direction of the shaft resistance is not immediate for the soft soil. Consequently, the amplitude of movement is larger but the global displacement in one cycle is smaller for soft soils.

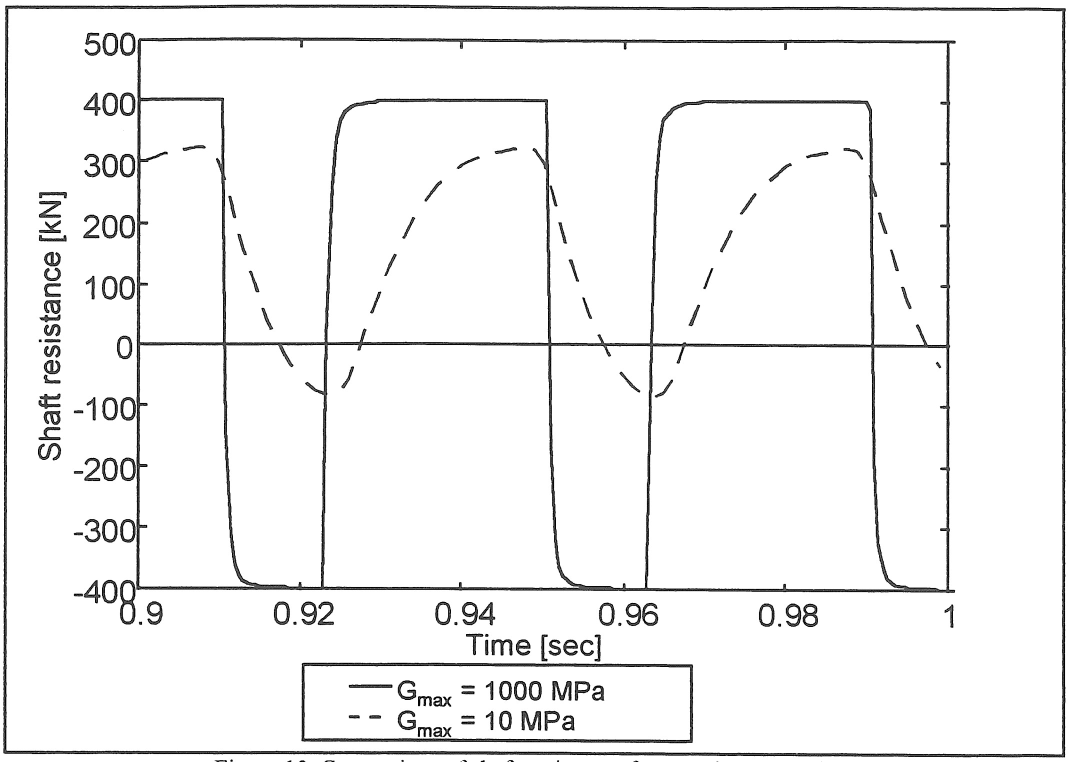

Figure 13: Comparison of shaft resistance for two shear modulus.

4. Conclusions.

Two different models to calculate the penetration speed have been developed and presented. These models are the first steps in the development of a more complicate and more realistic model to represent the sheet-pile movement during the vibratory driving.

The analysis of the results presented in this communication reports that the average penetration speed is higher for the hard soil than for the soft soil. The soft soil accompanies better the movement but the global displacement per cycle is smaller. For a given ultimate soil resistance, the perfect plastic model gives the highest speed.

The perfect plastic model allows to understand the different jumps in the acceleration curve. This observation suggests a new method to characterise experimentally the behaviour of soil. Indeed, the observation of the acceleration curve when the resistance is close to the action load would allow to show whether or not the soil has a plastic behaviour.

These two models give good solutions but consider drastic hypotheses. A more realistic model, should take into account the speed of load application, by introducing a visco-elasto-plastic soil behaviour. We must introduce the degradation of the soil resistance induced by the cyclic load. Indeed, that characteristic of soil is the most important during a vibratory driving. The build-up of pore pressure and the potential liquefaction are two important parameters that must be introduced in a model if it has to be realistic.

5. Acknowledge

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