Soil-structure interaction during pile vibratory driving

Interaction sol-structure lors du vibro-fonçage de pieux

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ABSTRACT : A rational procedure to model the dynamic nonlinear soil structure interaction during pile vibratory driving is presented. The procedure is based on the analysis of the dynamic behavior of a cylinder embedded in a semi- infinite medium. Cylindrical shear waves that propagate away from the vibrated pile are evaluated using a one-dimensional radial discretization of the soil surrounding the pile. Degradation of the skin friction upon cyclic shear stress is evaluated by applying elements of earthquake engineering practice used to assess liquefaction potential. The friction ratio as measured in a CPT test, serves as a correlation parameter to present charts allowing one to estimate the soil degradation and excess pore-pressure buildup. A sample calculation illustrates the importance to account for soil cyclic degradation.

RESUME : Une méthode rationelle pour modéliser l'interaction sol-structure dynamique non linéaire d'un pieu en cours de vibrofonçage est présentée. L'approche proposée s'appuie sur l'analyse du comportement d'un cylindre enfoui dans un milieu semi-infini. Les ondes cylindriques qui se propagent depuis le pieu vibré sont modélisées au moyen d'une discrétisation radiale du milieu continu entourant le pieu. La dégradation du frottement latéral suite à l'application des sollicitations cycliques s'estime au départ d'éléments développés en génie parasismique pour évaluer la liquéfaction des sols. L'indice de frottement déduit de l'essai de pénétration statique au cône est utilisé pour établir des abaques permettant d'évaluer la dégradation du sol ainsi que son degré de liquéfaction. L'exemple de calcul présenté illustre l'importance à tenir compte de la réduction cyclique du frottement latéral.

1 INTRODUCTION

A model aimed at representing the dynamic response of soils under cyclic loading induced by vibratory driving is presented. The source of the cyclic loading acting upon the soil is a pile or sheet pile being activated by a vibrator. As a first approximation the vibration pattern of the surrounding soil can be considered to possess cylindrical symmetry, assuming that the vibrations are essentially vertical.

The sheet pile is represented in this preliminary development by a rigid cylinder with a radius r_0 acted upon by the inertial effects of the vibrator eccentric masses, and deriving restrain from the dynamic reactions of the surrounding soil. The model used to represent the soil reactions is described below.

2 LOADING CONDITIONS AND MODEL GEOMETRY

In the case of a vibrating sheet pile, the governing boundary condition at the pile face should, in our opinion, be kinematic (i.e. velocity) rather than dynamic (i.e. stress). This approach differs from that followed in laboratory experiments where stresses are often used as the primary boundary condition. However the approach followed herein is supported by calculations performed with a single degree of freedom (SDOF) program (Holeyman, 1993a) that indicate a vibratory behavior (i.e. the amplitude of the movement) of the sheet pile itself not strongly influenced by the soil resistance. In soft soils the shear stress resisting the sheet pile movement is small, while it is higher in stiffer soils. It is therefore of interest to base the present soil model on strain-controlled cyclic shear tests rather than stress-controlled cyclic shear tests.

For the purpose of the present analysis, the soil reactions are separated into skin friction and toe resistance. Toe resistance is represented by a SDOF, commonly utilized in wave equation calculations (Holeyman, 1988). Because of its preponderance in the study of vibration and penetration of sheet piles, the skin friction is addressed by a more complex model that aims at encompassing the fundamental aspects of the vibratory behavior of the soil around the sheet pile.

The geometric shape of the proposed soil model surrounding the sheet pile or pile has cylindrical symmetry, as shown in Fig. 1. It is a disk with a thickness that slightly increases linearly with the radius. Normalized to the penetration depth h_0 of the sheet pile, it has a thickness h provided by the following equation:

$$h / h_o = [1+ 0.03 . (r-r_o)/r_o] \qquad (1)$$

The increase of the disk thickness with radial distance r tends to simulate the geometrical damping provided by the half-space of soil located below the toe of the sheet pile. The equivalent radius r_o of the sheet pile is obtained from perimeter considerations. The outer boundary of the model is set at a radial distance R_m based on a trade-off between calculation time and zone where the evaluation of the vibrations is of interest. An energy absorbing boundary condition in accordance with plane-strain elasticity theory (Novak et al, 1978) limits the lateral extent of the model at a distance large enough to ensure that deformations stay within the elastic range and to avoid artificial energy reflections.

The system of cylindrical waves propagating within the geometric model is calculated by discretizing the medium into

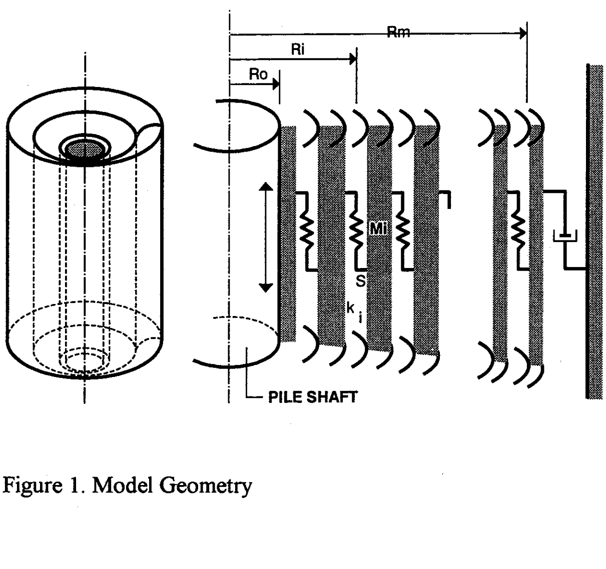

Figure 1. Model Geometry

concentric rings that possess individual masses and that transmit forces to their neighboring ones. The shear force-displacement relationship between successive rings is established based on the stress-strain relationship. Inter-ring reactions T_i are obtained based on ring displacements u_i using the following relationship:

$$T_i = 2\,\pi\; r_i\; .\, h_i\, .G'.\,(u_{i+1} - u_i) \qquad (2)$$

with G' representing the generalized secant shear modulus as discussed below. Movement of the rings is evaluated from the time integration of the laws of motion, and in particular from the acceleration resulting from the net unbalanced loads acting on each ring.

3 CONSTITUVE RELATIONSHIP

The constitutive relationship proposed for the representation of the large-strain, dynamic and cyclic shear stress-strain behavior of the medium surrounding the vibrating sheet pile is described by several laws addressing the following elements:

- Static stress-strain law expressing nonlinear behavior under monotonic loading and hysteresis upon strain reversal,
- Shear modulus at small strains and ultimate shear strength based on soil characterization: nature, void ratio, overconsolidation ratio or based on correlations with data obtained from CPT tests,
- Softening and increase of hysteretic damping with increasing strain depending on based on soil characterization,
- Effect of strain rate on initial shear modulus and ultimate strength,
- Degradation of properties resulting from the application of numerous cycles,
- Generation of excess pore possibly pressure leading to liquefaction and substantial loss of resistance, and
- Accommodation of variable strain amplitude history.

The following paragraphs address these components of the constitutive relationship.

3.1 *Static Stress-strain behavior*

A typical soil response to uniform cyclic strains with amplitude γ_c is represented in Fig. 2, which highlights or allows one to derive the following fundamental parameters:

G_{max}: initial (or tangent) shear modulus
τ_c : shear stress mobilized at γ_c
G_s: secant (or equivalent) shear modulus
λ : hysteretic (or intrinsic) damping ratio = $\Delta W / 2\pi\gamma_c\tau_c$, with ΔW = Energy lost in a given cycle

Both G_s and λ are strain-dependent parameters that need to be described by specific laws within a given cycle. τ_{max} is the ultimate shear strength, revealed at large strains. τ_{max} and G_{max} are shown to decrease with the number of cycles (cyclic degradation).

3.2 *Initial Shear modulus and ultimate shear strength*

Numerous studies have dealt with the initial shear modulus to be used in earthquake engineering (e.g. Drnevich et al., 1967). However, most of them are supported by parameters determined in the laboratory which are generally not available when a vibratory penetration issue arises. That is why the following, empirical approach based on correlations with CPT data (cone resistance q_c, local skin friction f_s, and friction ratio FR), is suggested :

$$G_{max} = K\,.\,q_c \qquad \text{with } K=15 \qquad (3)$$

$$\tau_{max} = \beta\,.\,f_s \text{ with } \beta = 0.65 + 0.35\,.\,\mathrm{Tanh}\;1.5\,(FR-2\,\%) \qquad (4)$$

3.3 *Secant Shear Modulus and Hysteretic Damping*

As can be observed in Fig. 2, G_s decreases with the shear strain during the initial monotonic loading. The curve that represents the initial monotonic loading is referred to as the initial "backbone" curve, because it also serves as the basis to generate the family of curves corresponding to unloading and reloading. Kondner's mathematical formulation (1963) is frequently employed to describe the initial backbone curve in earthquake engineering:

$$\eta = \tau / \tau_{max} = \delta / (\delta + 1) \quad \text{with} \quad \delta = \gamma / \gamma_r = \gamma\,.G_{max} / \tau_{max} \qquad (5)$$

It is of interest to represent that hyperbolic law in terms of reduced variables (η, the mobilization ratio and δ, the relative shear) as shown in Fig. 3. γ_r is called the reference strain. Two of the three parameters G_{max}, γ_r, and τ_{max}, are generally derived from laboratory experiments. In the case of CPT data, we propose to use τ_{max} per equation (4) and

$$\gamma_r = \tau_{max} / G_{max} = \beta.\; FR / K \qquad (6)$$

From the point of maximum straining, the unloading curve is described by the following equation, in accordance with Masing's rules 1 and 2 (Masing, 1926):

$$\tau - \tau_o = (\gamma - \gamma_o) / (1/G_{max} + (\gamma - \gamma_o)/2\tau_{max}) \qquad (7)$$

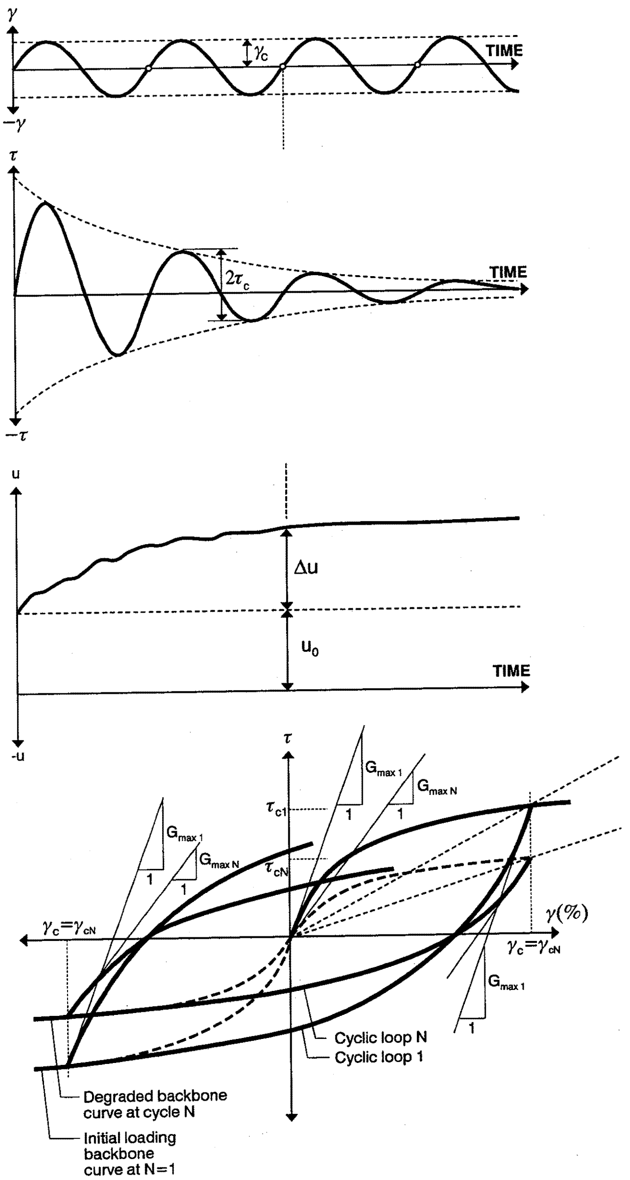

Figure 2. Soil Behavior under Constant Cyclic Shear Strain Amplitude Loading (From Vucetic, 1993; 1994)

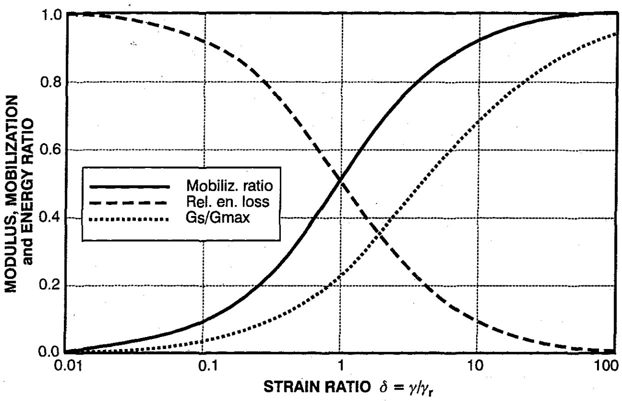

Figure 3. Hyperbolic law to evaluate G/Gmax and Relative Energy Loss

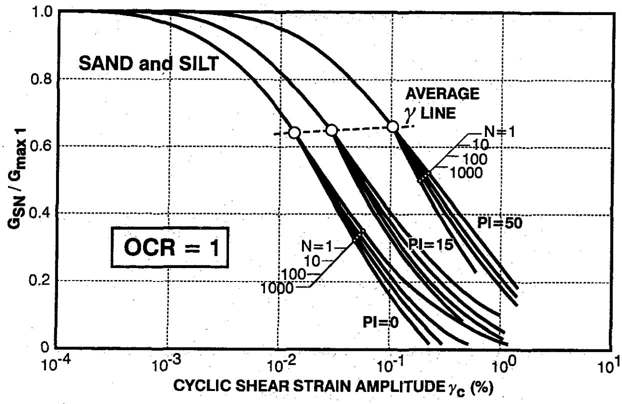

Figure 4. Soil stiffness degradation resulting from cyclic shear (Vucetic, 1993)

The energy contained in a loop depends for a given soil on the amplitude of the cyclic strain. Empirical data collected in laboratory tests indicates that the damping ratio increases with γ_c as the soil undergoes higher plastic deformations.

The unifying approach developed by Dobry and Vucetic (see Dobry and Vucetic 1987, Vucetic and Dobry, 1991, and Vucetic,1993 and 1994 is applied to accommodate the influence of the nature of the material characterized by the plasticity index (PI), as indicated in Fig. 4. The relative energy loss (or π x damping ratio) as obtained by directly integrating the area defined by a loop in the stress-strain diagram has also been represented in Fig. 3 to demonstrate the ability of the hyperbolic law to reproduce experimental observations. The PI influencing the value of the reference strain γ_r available from other studies was correlated to the friction ratio using:

$$PI = 50\,.(1 + \mathrm{Tanh}\,(FR - 3.5\%)) \quad (8)$$

3.4 *Strain Rate Effects*

Although it is well known that undrained modulus and shear strength increase with increasing strain rate ($\dot{\gamma}$), experimental data generated using different apparatuses and loading conditions lead to different conclusions. Viscosity mechanisms may well provide a suitable framework to understand the strain rate effect observed when comparing fast and slow undrained monotonic stress-strain curves, as well as to explain the roundness of the loop tips during a sinusoidal strain-controlled cyclic test. Evidence would point to the fact that sands and non plastic silts have very small viscosity in that their stress-strain loops exhibit sharp rather than rounded tips (Dobry and Vucetic, 1987).

The mathematical functions proposed in the literature to represent the nonlinear viscosity also depend on the type of experimental observations. We propose to adopt a power law:

$$\tau_{vel} = \tau_{sta}\,.\,(1 + J\,.\,\dot{\gamma}^n) \quad (9)$$

The advantage of that mathematical form is that resistance does not become zero at zero strain rate. The power law also requires the strain rate to vary by orders of magnitude to provide tangible increases in both the modulus and the ultimate strength. The J coefficient and n exponent depend on the nature of the soil. Based on pile driving data, we propose to use n=0.2 and J=0.3 $s^{-0.2}$ for plastic soils. J should therefore essentially depend on the plasticity of the soil, and thus on the FR obtained from CPT tests, as proposed in first approximation below:

$$J = 0.10\,FR \quad (10)$$

3.5 *Degradation Law*

When subjected to undrained cyclic loading involving a number N of large strain cycles, the soil structure continuously deteriorates, the pore pressure increases, and the secant shear modulus decreases with N. This process known as cyclic stiffness degradation can be best characterized on the basis of strain controlled tests for the type of loading involved with the vibratory penetration of sheet piles. Typical results of strain-controlled tests are sketched in Fig. 4, where the degradation is clearly expressedby the decrease of the amplitude of the peak stress mobilized at successive cycles.

The quantification of the degradation process calls for the introduction of the degradation index Δ, defined by:

$$\tau_n = \Delta.\,\tau_1 \quad (11)$$

Laboratory results conducted at constant cyclic strain show that in many soils, the degradation index can be approximated by the following relationship as suggested by Idriss et al (1978):

$$\Delta = N^{-t} \quad (12)$$

The exponent t, called degradation parameter, depends mainly on the amplitude of the cyclic strain and the nature of the material (PI), as suggested by Dobry and Vucetic (1988) and as indicated in Fig. 5 (Vucetic, 1993). It is noteworthy that the degradation parameter assumes a zero value at strains smaller than a cyclic "threshold" shear strain,$\gamma_{\tau\upsilon}$ The threshold strain increases with the plasticity of the soil, as suggested in Fig. 5.

Laws to represent the degradation based on CPT results are proposed as follows:

$$\gamma_{\tau\upsilon} = \beta.\,FR/30 \quad (13)$$

$$t = (\gamma/\gamma_{\tau\upsilon} - 1)^{1/2} / (PI/2 + 25) \quad (14)$$

3.6 *Soil liquefaction*

Vibration induced compaction of saturated sands has received attention not only from the earthquake engineering community, but also from vibro-compaction specialists. Recent advances tend to indicate that build up of pore pressures (eventually leading to liquefaction) and volume reduction of cyclically loaded materials are the expression of the same phenomenon, i.e. the irreversible tendency for a particulate arrangement to achieve a denser packing when sheared back and forth.

Under drained conditions, the volume reduction is immediate. Under undrained conditions, the tendency for volume reduction is

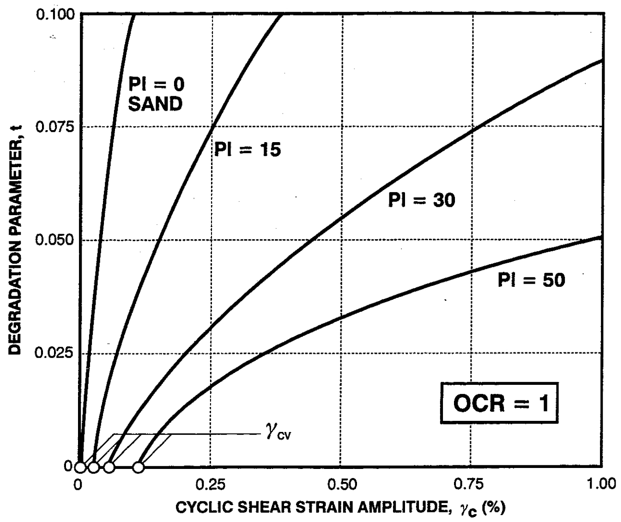

Figure 5. Effect of Plasticity Index (PI) on soil degradation (Vucetic, 1993)

expressed by an increase in the pore water pressure (see Fig. 2), such that the effective stress is reduced to a value that may be close to zero. It is then necessary to wait for the sample to consolidate in order to see the volume reduction take place.

The strain driven evaluation of the build up of pore pressure as suggested by Dobry et al. (1979) has been adopted as it leads to a direct transposition to the problem of the vibrations induced by a vertically vibrating sheet pile. It also allows one to evaluate potential changes of the void ratio based on a cyclic strain rather than stress history, as evidenced in the early tests conducted on drained sands by Youd (1972). Finally, this framework of analysis enables the threshold cyclic strain to encompass in a single concept the intrinsic relationship between degradation and pore pressure build-up, with the advantage that it can be applied to general categories of soils (sands to clays)

The excess pore pressure generated during cyclic loading has been shown (see Fig. 6) to increase with the shear strain and the number of cycles for a given soil type. We have adopted the damage parameter κ approach (Finn, 1981) to evaluate the excess pore pressure δu resulting from a particular strain history, as characterized by the following equations:

$$\delta u/\sigma' = \tfrac{1}{4}\,(\text{Rel. En. Loss}) \,.\, \ln(1 + \tfrac{1}{2}\kappa) \qquad (15)$$

with Relative Energy Loss given by Fig. 3, and κ = damage parameter given by:

$$\kappa = \xi\, e^{\Lambda\gamma} \text{ with } \quad \xi = \text{length of strain path} \qquad (16)$$
$$= 4\,N\,\gamma_C, \text{ for constant amplitude cycles}$$
$$\Lambda = 5$$

3.7 *Generalized strain history*

Because the constitutive relationship parameters are generally established on the basis of constant strain, laboratory controlled tests, it is necessary to formulate a means to follow the dynamic behavior of the medium under the non regular types of loading present during the vibratory penetration of a sheet pile: start-up and turn-off, or progressive modification of soil properties resulting from degradation. Masing rules 3 and 4 have thus been applied to accommodate non-repetitive straining paths.
Degradation can also be represented under irregular loading, provided that a degraded backbone curve is used instead of the initial backbone curve. The degraded backbone curve iscompletely defined by the degradation index Δ, the equivalent value of which must be ascertained based on the amplitude and number of previous straining cycles. This will require the updating of the current degradation index using the following equations:

$$N_{eq} = \Delta^{(-1/t)} \qquad \text{and} \qquad \Delta_{new} = (N_{eq} + 1)^{-t} \qquad (17)$$

4 MODELING RESULTS

4.1 *Constitutive law*

The CPT has been chosen as the basic sounding upon which the evaluation of the vibratory penetration of sheet piles is to be conducted. It provides simple, yet ad hoc parameters that can be related directly or indirectly to the parameters necessary to model the soil behavior. Because the proposed approach is based on desk generated correlations, full scale tests will be used to refine the correlations by matching the penetration speeds and vibration levels calculated by the model with those measured in the field.

At this stage of reporting of the development of the model, the reasonableness of the proposed constitutive law and correlation can be ascertained by graphically representing numerical results derived from our comprehensive set of assumptions. Samples of those graphical results are presented in Figs 7, 8, and 9. Comparison of Fig. 7 with Fig. 5, Fig. 8 with Fig. 4, and Fig. 9 with Fig. 6, respectively, provides a measure of the preliminary

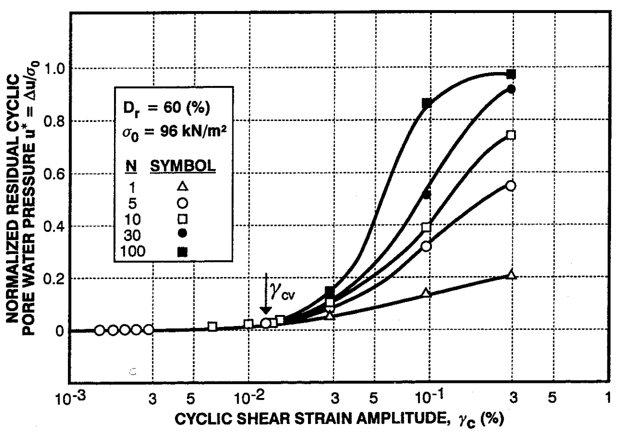

Figure 6. Build up of residual pore pressure in different sands in undrained cyclic triaxial strain-controlled tests (Dobry et al, 1982)

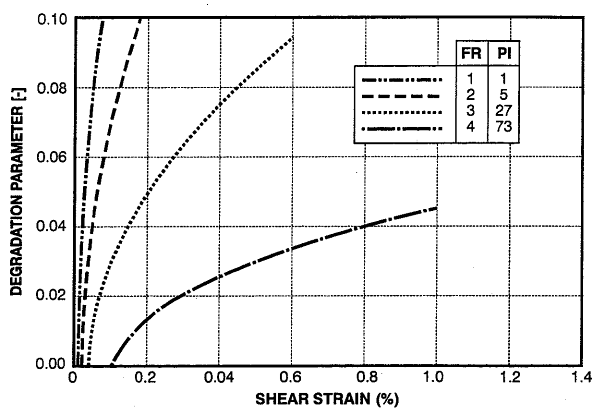

Figure 7. Modelled degradation Parameter versus Shear Strain for different Friction Ratios

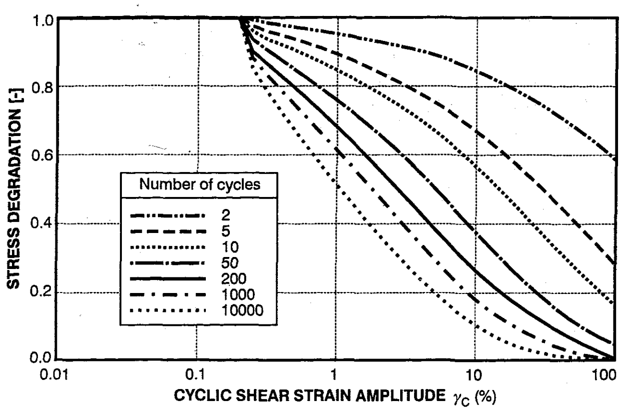

Figure 8. Modelled degradation versus Strain (Friction Ratio = 2%)

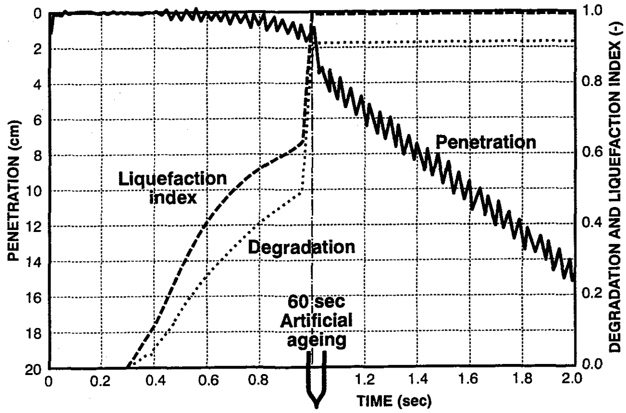

Figure 10. Penetration vs time

satisfactory agreement between the general trends depicted by the constitutive law proposed herein and available laboratory results.

4.2 *Example*

Results of a sample problem are discussed herein and illustrated in Figs 10 and 11. The example provided is characterized by the following parameters :

Vibrator

Excentric moment	=	50	kg.m
Frequency	=	27	Hz
Vibrating mass of vibration, including champ	=	6,700	kg
Stationary mass of vibration	=	3,500	kg

Sheet Pile (steel)

Section	=	167	cm^2
Perimeter	=	2.88	m
Length	=	11.7	m
Penetration	=	10	m

Soil Conditions

Depth to water table	=	2	m
Average come resistance along slaft	=	6	MPa
Average friction ratio along shaft	=	2	%
Core resistance at the toe level	=	10	MPa
Friction ratio at the toe level	=	2	%

Calculations were conducted using model comprizing twenty radial elements and reaching an outer radius of 2 m. The vibrator frequency is increased from 0 to 27 Hz during the first ½ second and maintained constant thereafter. An artificial ageing is activated after one second in order to reduce the computation time : the equivalent number of cycles during an ageing period of 60 seconds is used to evaluate the regime degradation index, according to equation (12). The more on less stabilized penetration regime is then simultated for an additional second. The average penetration speed is estimated from that later portion of the simulation.

Fig. 10 clearly slows the effects of degradation above and below the water table as time progresses and ageing is activated. Fig. 11 presents a detailed view of the force diagrams covering the last 0.1 s of the simulated period. It can be noted that the velocity (multiplied by the pile mechanical impedance = 0.678 MN/ms^{-1}) is quasi-sinusoidal. On the other land, the toe resistance is only activated when the velocity and net displacement are positive. The skin friction alternates between positive and negative phases, with a longer duration in the positive phase, as can be expected from a net penetrative movement.

5 CONCLUSIONS

A constitutive relationship governing the nonlinear cyclic behavior of soil under cylindrical shear as imposed by sheet pile vibratory driving has been developed and presented. The suggested relationship highlights degradation of the soil resistance under cyclic loading as a key parameter in modeling vibratory

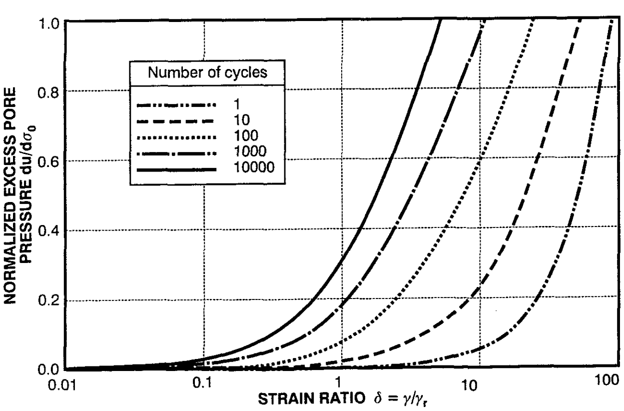

Figure 9. Modelled pore Pressure versus Relative Strain for different Numbers of Cycles

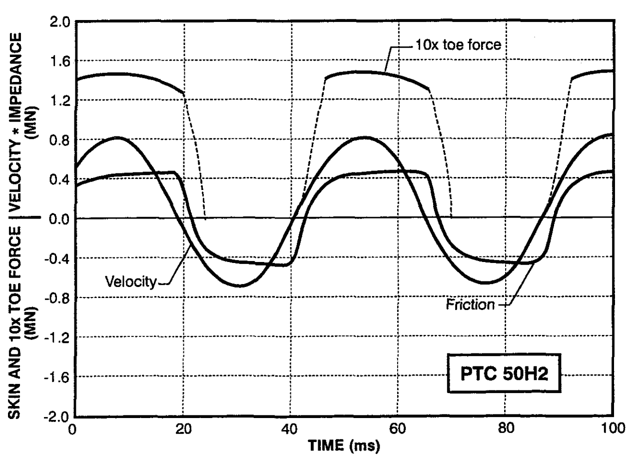

Figure 11. Pile velocity and soil resistance

penetration. Confirmation of the proposed correlations between constitutive parameters and results of CPT tests requires incorporating the constitutive relationship into a radial discrete model and the comparison of calculated physical features, such as penetration speed and ground vibration levels, with those measured during full scale penetration tests.

6 ACKNOWLEDGMENTS

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