

APPLICATION OF FLOW EQUATION IN UNSATURATED SOIL TO A SELECTIVE SORBENT.

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ABSTRACT

This paper investigates the possibility to extend the models used frequently in soil mechanics to another materials. The absorption of fuel oil by a man-made sorbent is compared with the absorption of water by the soil, a natural material.

A synthetic sorbent with capillary properties allowing to separate water from oil products was investigated. The sorbent is made of a dense network of intertwined fibers.

Laboratory absorption tests were performed on a column of this material using fuel oil. The experimental set-up allowed to establish the evolution of the absorption as function of the time. The absorption curve observed at equilibrium had the same characteristics as the water absorption curve observed for a soil.

The experimental set-up was modelled using a numerical scheme integrating the equation of the water flow equation in an unsaturated soil. The two constitutive descriptions needed by the model were the absorption curve and the conductivity curve. The absorption curve was represented with the empirical model of van Genuchten whose parameters were calculated on the absorption curve measured at equilibrium. The conductivity curve was calculated by using the model of Mualem whose parameters depend of those needed for van Genuchten's model. The different transient stages observed during the experiment before matching equilibrium were also calculated. A very good accordance between the calculated results and the experiments was observed.

The results of this investigation show that models frequently used to represent the water flow in unsaturated soil can be extended to other materials, even if the structure of the materiel is very different.

DESCRIPTION OF THE SYNTHETIC SORBENT

The sorbent used in this investigation is a synthetic foam that absorbs selectively a large set of pollutants (a.o. products extracted from the petroleum industry, alcohol, solvent, ...).

The foam is produced in two steps. The first step is the chemical reaction of the product that constitutes the foam. During this reaction, the foam inflates and reaches a very high porosity (approximately 98%). However, the structure thus produced is closed, made of gas bubbles imprisoned in a solid structure (figure 1). Therefore, no fluid is able to move through the structure. The second step of the production of the foam is a mechanic compression whose the effect is to break down the closed structure. This compression opens the communication between the bubbles and allows the migration of fluids (figure 2). After this operation, the porosity of the sorbent is reduced to approximately 93%.

The characteristics of the sorbent are summarised in the table 1.

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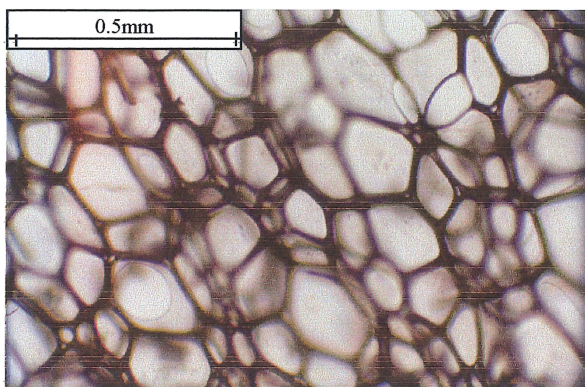


Figure 1 : Structure of the foam after reaction

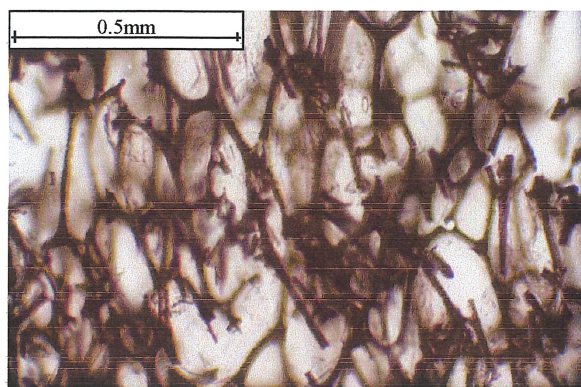


Figure 2 : Structure of the foam after compression

Volumetric mass	80	kg/m ³
Total Porosity	93	%
Volumetric mass of the fibbers	1262	kg/m ³
Swelling during fuel oil absorption	0 to 12	%
Fuel oil conductivity in saturated medium	$1.6 \cdot 10^{-4}$	m/s

Table 1 : Sorbent characteristics

The capillary characteristics of this sorbent enable it to separate water from the other products. Therefore, if a piece of foam is placed in a bowl containing water and fuel oil (figure 3), only fuel oil will be absorbed. The fuel will completely saturate the sorbent volume below the oil surface but also oil will be absorbed by capilarity in the volume above the oil surface. Experiments have shown that the oil content progressively decreases as the distance above the oil surface increases.

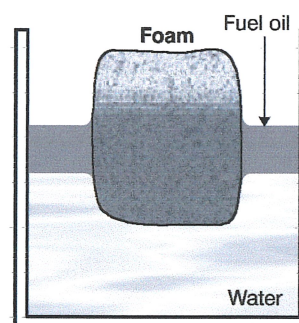


Figure 3 :Selective sorbent

FUEL OIL ABSORPTION

Experiments were performed to characterise the sorbent capacity to absorb the fuel oil by capilarity by observing the evolution of the absorption in time and space.

The test (Figure 4) consists in placing the lower extremity of a column of foam in fuel oil during a set period. When a given time of absorption is reached, the column is cut in small pieces. The fuel oil content of each piece and the height of the piece above the oil surface are then measured. Representing the measurements of the oil content of each piece as a function of the height of the corresponding piece, it is possible to determine the absorption curve of the fuel oil in the foam for the time chosen for the test. Resuming this procedure for different absorption times, it is possible to determine the evolution of the absorption as a function of time (figure 5).

The experimental results show the progression of a front line that progressively goes to the equilibrium. The absorption curve at the equilibrium has a shape similar to the curves measured for a soil with an air entry point equal approximately to 120mm. The evolution of the fuel oil content below the air entry point is the result of two phenomena that take place simultaneous: first, the tortuosity of the foam delays the total saturation and, secondly, the volume of the sorbent increases progressively and can reach 112% of the initial volume.

The absorption curve at the equilibrium (Figure 6) can be described with the van Genuchten equation (1980). This empirical equation is traditionally used to characterise the absorption curve in soil:

$$\theta_E = \frac{1}{1 + \left(\alpha h \right)^{\frac{1}{1-m}}^m} \quad (1)$$

where θ_E is the degree of saturation
 h the height above the fuel oil surface in the reservoir [mm]
 α and m are empirical parameters

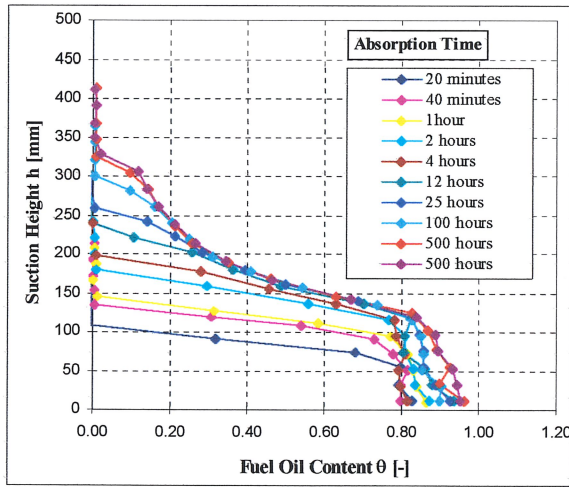


Figure 5 : Evolution of fuel oil absorption.

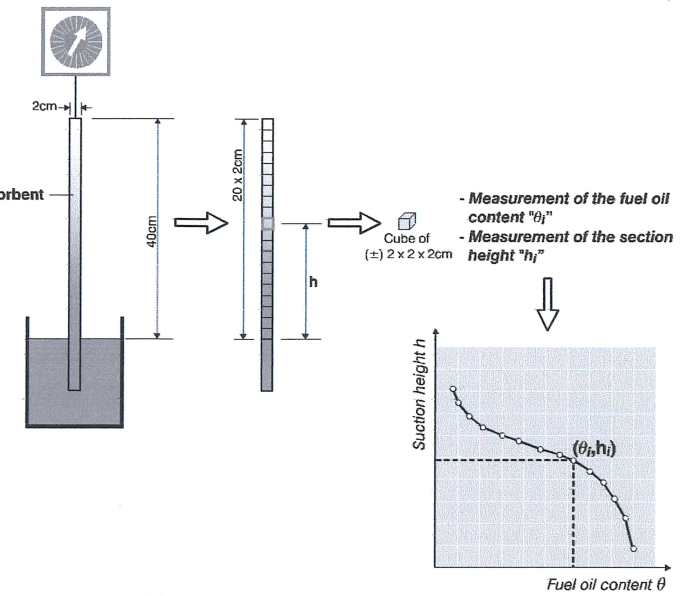


Figure 4 : Experimental set-up.

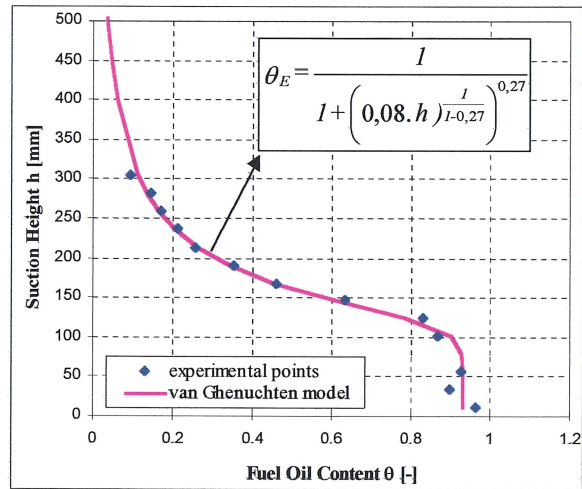


Figure 6 : Modelisation of the absorption curve with the van Genuchten model

FLOW EQUATIONS IN UNSATURATED SOIL.

The flow of water in a unsaturated column of soil can be modelled with the well-known equation of Richards (1931):

$$c(h) \frac{\partial h}{\partial t} = \frac{\partial}{\partial z} \left(k(h) \frac{\partial h}{\partial z} \right) + \frac{\partial k(h)}{\partial z} \quad (2)$$

where $c(h)$ is the specific capacity of retention [m^{-1}]
 $k(h)$ is the conductivity curve [m/s]
 z is the vertical axis [m]
 $h(z,t)$ is the suction applied at the height z and the time t [m]

The specific capacity of retention $c(h)$ is defined as the slope of the retention curve and can be described with an equation deduced from the van Genuchten equation:

$$c(h) = \frac{d\theta}{dh} \quad (3)$$

The conductivity curve $k(h)$ represents the evolution of the conductivity when the water content of the soil decreases under the effect of the suction h . Indeed, when a suction h is applied on a soil, the water content, and therefore the degree of saturation θ_E decreases. Therefore, the number of pores that are able to participate to the flow decrease simultaneously.

Mualem (1976) proposed a method to calculate the conductivity curve based on the absorption curve. In Mualem's model (figure 7), pores participate to the flow only if the size of the pore is small enough to be filled at the suction h : when the soil is completely saturated, all the pores participate to the flow and the conductivity is maximal. When the suction increase, the water content decreases following the absorption curve. The decrease of the number of pores participating to the flow results in a decrease of the conductivity. The model calculates the flow $V_p(r_i)$ that goes through each pore with a radius r_i using the Poiseuille equation. Based on the pore size curve $f(r_i)$, the flow q_i that goes through all the pores with a radius r_i is equal to $f(r_i) \cdot V_p(r_i)$. The total flow q that goes through the section of soil under a suction h is the sum of all the flows q_i of each class of pores that are filled under the suction h :

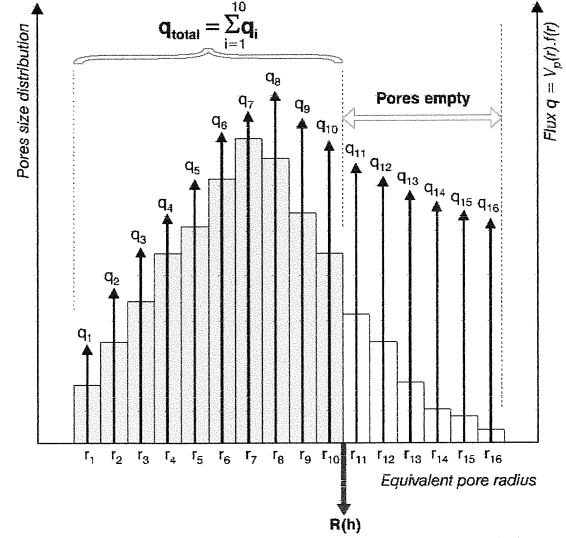


Figure 7 : Principle of Mualem model.

$$q = \sum_i q_i = \frac{1}{\tau} \int_0^{R(h)} V_p(r) f(r) dr = k(h) \frac{i}{\sum \pi \cdot r_i^2 \cdot f(r_i)} \quad (4)$$

- where r is the radius of the pore [m]
- τ is the tortuosity defined as the ratio between the distance in straight line and the distance really followed by the fluid [-]
- $V_p(r)$ is the flow that goes through a pore with a radius r [m/s]
- $f(r)$ is the number of pores with a radius r [-]
- $R(h)$ is the size of the biggest pores that is still filled under the suction h [m]
- i is the hydraulic gradient [-]

The absorption curve is calculated using the empirical equation of van Genuchten (eq. 1) and the pore size curve is deduced from this absorption curve.

A correction is introduced in the equation 4 to take into account the increasing of the tortuosity when the water content decrease. This phenomena is described with the exponential model of Carey:

$$\tau = \tau_s \cdot \theta_E^{-b} \quad (5)$$

- where τ_s is the tortuosity in saturated condition [-]
- θ_E is the degree of saturation [-]
- b is a empirical parameter [-]

Using van Genuchten equation and Carey's model, the expression 4 can be developed and the conductivity is calculated with the expression (Mualem, 1976):

$$k(\theta_E) = k_{sat} \theta_E^b \left[1 - \left(1 - \theta_E^{1/m} \right)^m \right]^2 \quad (6)$$

- where k_{sat} is the conductivity in saturated condition [m/s]
- θ_E is the degree of saturation [-]
- b, m are empirical parameters [-]

In this equation, the exponent b describes the tortuosity of the soil and is generally equal to 0.5 for the soil. The parameter m is deduced from the measured absorption curve at the equilibrium modelled with the van Genuchten model. The Mualem model allows to integrate the Richards equation by knowing only the absorption curve and the conductivity in saturated condition.

MODELISTATION OF THE ABSORPTION OF FUEL OIL IN THE SORBENT

The Richards equation combined with the Mualem model was used to calculate the evolution of the absorption of fuel oil in a band of foam used in the experiments described above. The differential equation of Richards was integrated with the finite difference method using the numerical code WAVE. The empirical parameters α and m of the van Genuchten equation were chosen to fit the absorption curve measured near the equilibrium (Figure 6). The conductivity in saturated condition was measured in a permeameter ($k_{sat} = 1.6 \cdot 10^{-4}$ m/s).

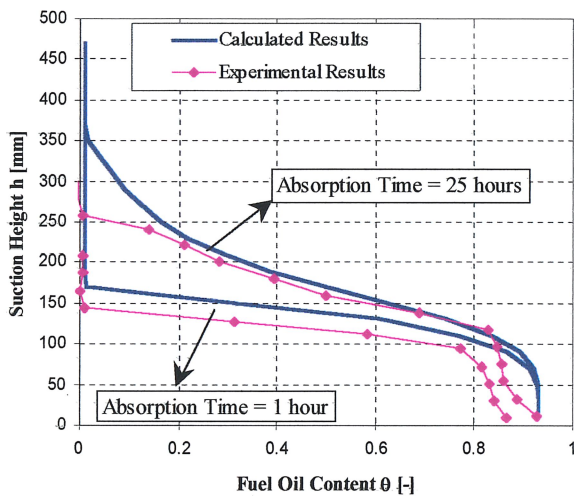


figure 8 : Comparison between calculated and experimental results for $b=0.5$.

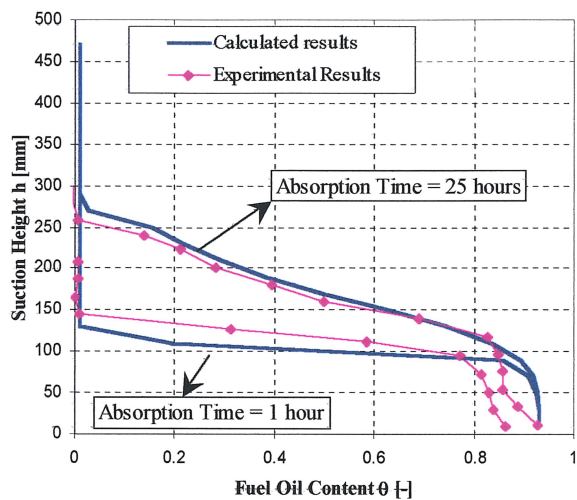


figure 9 : Comparison between calculated and experimental results for $b=1.3$.

The result of the calculation for 2 different absorption times (40 minutes and 4 hours) is compared with the experimental results on figure 8. The poor fit between the model results and the experiments is probably caused by the difference in the structure between the foam and the soil. Indeed, the soil is a granular material whereas the foam is a set of crushed “shells” intertwined together. Therefore, the tortuosity is more important in the foam than in a soil and, in particular, when the fuel oil content decreases under the effect of the suction. The parameter b that describes the tortuosity in the Mualem model is increased and it was found that for a value of 1.3 (instead of 0.5 for the soil) the model is able to represent correctly the evolution of the absorption. Figure 9 compares the results of the calculation with the experiments and Figure 10 shows the evolution of the absorption.

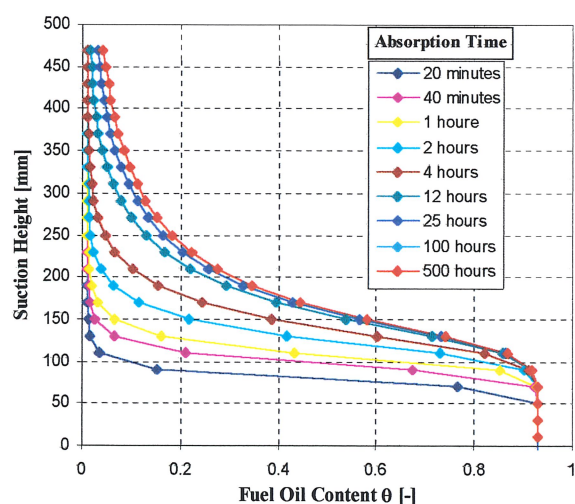


Figure 10 : Calculated absorption curves.

CONCLUSION

The present paper presents the result of a investigation where equations developed for a granular material have been applied to an artificial sorbent. The sorbent is not a granular material but a set of "shells" intertwined together: it is like a "anti" soil. It was shown that the Richards equation combined with the Mualem model is able to represent the evolution of the absorption of fuel oil by capilarity in a column of foam. Only the parameter that represents the tortuosity had to be increased to take into account the differences in the structure between the soil and the foam.

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NOTATIONS

h	Height above the fluid, suction height
t	Time
z	Vertical axis
θ	Volumetric water/ fuel oil content
θ_E	Degree of saturation
$c(h)$	Specific retention capacity
α, m	Empirical parameters of the van Genuchten equation
q	Flow
k_{sat}	Conductivity in a saturated media
$k(h)$	Conductivity under a suction h
τ	tortuosity
τ_s	tortuosity in a saturated media
b	Empirical parameter of the Carey model

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