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Application of a hypoplastic constitutive law into a vibratory pile driving model

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ABSTRACT: This paper describes a model calculating the penetration speed of a pile during its vibratory driving and the vibrations induced around it. The model is called VIPERE, for VIbratory PEnetration REsistance.
The model actually implements hypoplastic constitutive behaviour into a geometric model suggested by Holeyman (1994). The model VIPERE considers the pile as a rigid body and simulates the soil by a 1D radial discretisation. The soil behaviour is assumed to be hypoplastic and modeled using the Bauer (1996) and Gudehus (1996) constitutive law. The penetration speed and the wave propagation around the pile are evaluated by integrating the equation of movement.
The paper describes the model by specifying the assumptions relative to the pile and the soil behaviour, the equations used to evaluate the soil resistance along the pile shaft and at the pile base, and finally the procedure of integrating the motion equations. Typical simulations of pile driving are presented and discussed.

1 INTRODUCTION

The VIPERE model calculates the penetration speed of a pile or sheet-pile at a given depth during a vibratory driving.

The model considers the pile as a rigid body and represents the soil with a cylindrical discretisation of rings. The interaction between these rings is described by a hypoplastic constitutive law (Fig. 1).

To calculate the displacement of the pile as well as the wave propagation around the vibrating profile, the model integrates the equation of motion step by step. The acceleration is calculated at each time step, based on the balance of the forces acting on the pile and on each ground element considered in the discretisation.

The forces acting on the pile are:

- the vibrating force induced by the vibrator (= me.ω^2.sin(ωt) where me is the eccentric moment and ω is the angular frequency (= 2.π.Freq)),
- the static weight placed above the vibrator and isolated from it by shock absorbers (= M_{total} . g),
- the friction resistance along the shaft of the pile F_{shaft},
- the toe resistance F_{toe} and
- the inertial force induced by the movement of the mass of the pile and the vibrator.

The forces acting on each ground element of the discretisation are, in addition to the inertial force, the forces of friction generated on the internal and external faces by the movement of the close elements.

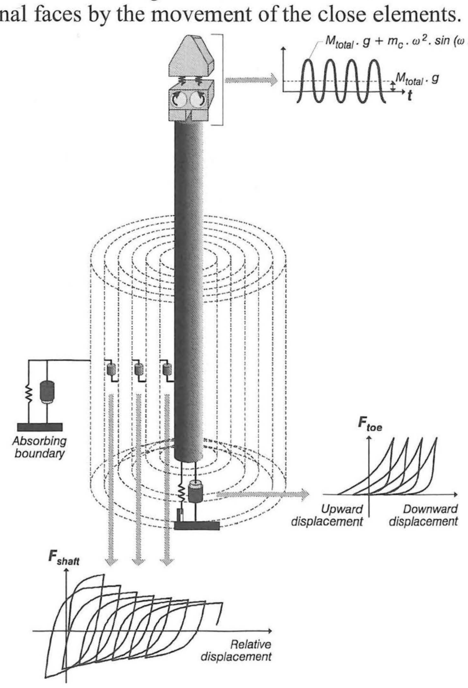


Fig. 1: Vibrodriving model.

During simulations of the vibratory driving of sheet-piles, the model does not take into account the influence of the sheet-pile wall formed by the other already installed sheet-piles neither the friction resistance generated at the interlock with the neighbour sheet-piles.

The scope of this paper is to describe the main assumptions considered in the VIPERE model and to show how the hypoplastic constitutive equation is implemented into the model. The integration procedure is also presented. Finally, the model performances are discussed through the analysis of simulation results.

2 GEOMETRIC MODEL

This model borrows the geometric configuration of the HYPERVIB II model developed by Holeyman (1994, 1996).

The geometric shape (Fig. 2 a) assumes the pile or sheet-pile and the surrounding soil have cylindrical symmetry. Since the shape of the pile is not necessary cylindrical, the equivalent radius of the pile is obtained from perimeter considerations. The soil is assumed to be a disk (Fig. 2 b) with a thickness that slightly increases in a linear way with the radius (Fig. 2). This increase tends to simulate the geometrical damping provided by vertical diffusion of waves around the profile.

$$\Delta h = \alpha . \Delta r \qquad \text{(Eq. 1)}$$

where r is radius and α is the coefficient of dispersion (= 0.03 – Holeyman, 1996)

In order to simulate the wave propagation, the soil is discretised into a set of concentric rings possessing individual mass and transmitting forces to their neighbouring ones (Fig. 2 c). The base resistance of each element is supposed to balance the gravity force. The vertical shear stress τ between two rings is calculated based on the relative displacement of each ring using the hypoplastic model (Gudehus, 1996; Bauer, 1996). The radial discretisation is characterised by the number of rings (Nr) and the maximum radius (R_{max}).

The boundary condition of the cylindrical discretisation is selected in such way that arriving waves are absorbed by that border (Novak, 1978). However, considering Novak's assumptions, this border will absorb the incident wave only if the wave induces a displacement of the soil element that stays in the linear elastic domain of the soil. In order to respect that condition, the number of soil elements and the maximum radius must be large enough to ensure the wave is damped enough at the boundary and is in the elastic domain.

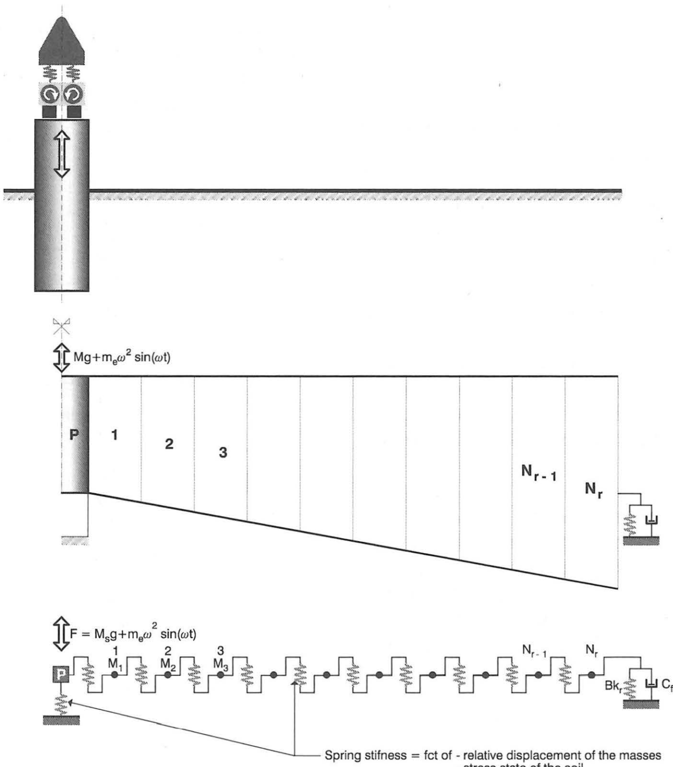


Fig. 2 (a,b,c):Geometric model. (After Holeyman, 1996)

3 CONSTITUTIVE LAW FOR SHAFT RESISTANCE

3.1 Assumptions

The model considers each soil element as purely sheared (Fig. 3). A shear displacement is imposed between the internal and external shaft of the soil element. Neither axial nor radial normal strains are permitted along these boundaries. The non-vanishing strain of the strain tensor is the shear strain γ_{rz} ($\gamma_{rz} \neq 0$; $\varepsilon_r = \varepsilon_\theta = \varepsilon_z = 0$).

The soil is assumed to be fully saturated and the frequency of the vibrator is supposed high enough to prevent excess pore water dissipation during the vibratory driving. Therefore, the model considers the behaviour of the soil is undrained (i.e. no volume change; $\Delta e = (1+e).(\varepsilon_r+\varepsilon_\theta+\varepsilon_z) = 0$).

It is assumed that there is no variation of the stresses and strains along the z axis ($\partial(..)/\partial z = 0$). The radial normal stress and the shear stress do not change as a function of the depth.

Based on these assumptions, Fig. 4 shows the stresses and strains distribution at each soil element interface. The shear strain γ_{rz} is calculated based on the relative displacements of these elements assuming a linear distribution of the displacement.

The resulting stress tensor T_s and strain tensor D_s can be written as follow[1]:

$$T_s = \begin{pmatrix} -\sigma_r' & 0 & \tau_{rz} \\ 0 & -\sigma_\theta' & 0 \\ \tau_{rz} & 0 & -\sigma_z' \end{pmatrix} \qquad D_s = \begin{pmatrix} 0 & 0 & \gamma_{zr} \\ 0 & 0 & 0 \\ \gamma_{rz} & 0 & 0 \end{pmatrix} \qquad \text{(Eq. 2)}$$

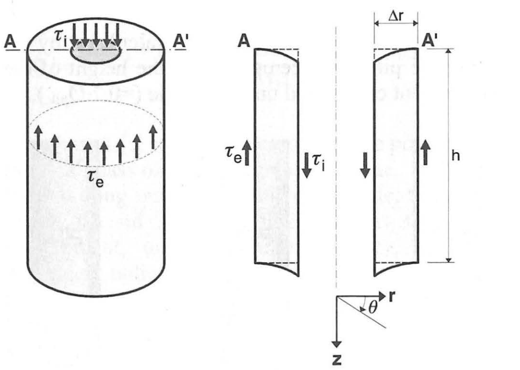


Fig. 3: Simple shearing in axisymetry

[1] For the hypoplastic constitutive equation, the compressive stresses and shortening strains are defined negative in accordance to the convention in continuum mechanics. Only the scalar of the effective mean stress P' ($= -\mathrm{tr}(T_s).\frac{1}{3}$) is defined positive in compression.

3.2 Hypoplastic model in cylindrical simple shearing

The constitutive relationship used to calculate the shear stress along the shaft of the different elements is the hypoplastic model (Kolymbas, 2000, Gudehus, 1996, Bauer, 1996). This constitutive law evaluates the stress rate tensor $\dot{T}_s$ as a function of the current stress state tensor T_s and the strain rate tensor $\dot{D}_s$ based on the following incremental equation:

$$\dot{T}_s = f_s . \left[a_1^2 . \dot{D}_s + \hat{T}_s . \mathrm{tr}\left(\hat{T}_s . \dot{D}_s\right) + f_d . a_1 . \left(\hat{T}_s + \hat{T}_s^*\right) . \left\|\dot{D}_s\right\|\right] \qquad \text{(Eq. 3)}$$

a_1 is a dimensionless scalar depending of the friction angle φ' of the soil, $\hat{T}_s = T_s/\mathrm{tr}(T_s)$ is the granular stress ratio tensor, $\hat{T}_s^* = \hat{T}_s - \frac{1}{3}.\mathbf{1}$ is the deviatoric part of $\hat{T}_s$, f_s and f_d are scalars depending of the mean stress P'_s and the void ratio e.

Based on notations and assumptions of paragraph 3.1, equation 3 can be simplified and can be expressed in its incremental shape into the following components of the stress state tensor:

$$\Delta\tau_{rz} = f_s . \left|\Delta\gamma_{rz}\right| . \left[\left(a_1^2 + \frac{2.\tau_{rz}^2}{(\sigma_r'+\sigma_\theta'+\sigma_z')^2}.\right) \mathrm{sign}(\Delta\gamma_{rz}) + f_d . \sqrt{2} . a_1 . \frac{2.\tau_{rz}}{.(\sigma_r'+\sigma_\theta'+\sigma_z')}\right]$$

$$\Delta\sigma_r' = f_s . \left|\Delta\gamma_{rz}\right| . \left[\frac{2.\tau_{rz}.\sigma_r'}{(\sigma_r'+\sigma_\theta'+\sigma_z')^2} . \mathrm{sign}(\Delta\gamma_{rz}) + f_d . \sqrt{2} . a_1 . \frac{(5.\sigma_r'-\sigma_\theta'-\sigma_z')}{3.(\sigma_r'+\sigma_\theta'+\sigma_z'')}\right]$$

$$\Delta\sigma_\theta' = f_s . \left|\Delta\gamma_{rz}\right| . \left[\frac{2.\tau_{rz}\,\sigma_\theta'}{(\sigma_r'+\sigma_\theta'+\sigma_z')^2} . \mathrm{sign}(\Delta\gamma_{rz}) + f_d . \sqrt{2} . a_1 . \frac{.(5.\sigma_\theta'-\sigma_r'-\sigma_z')}{3.(\sigma_r'+\sigma_\theta'+\sigma_z')}\right]$$

$$\Delta\sigma_z' = f_s . \left|\Delta\gamma_{rz}\right| . \left[\frac{2.\tau_{rz}\,\sigma_z'}{(\sigma_r'+\sigma_\theta'+\sigma_z')^2} . \mathrm{sign}(\Delta\gamma_{rz}) + f_d . \sqrt{2} . a_1 . \frac{.(5.\sigma_z'-\sigma_r'-\sigma_\theta')}{3.(\sigma_r'+\sigma_\theta'+\sigma_z')}\right] \qquad \text{(Eq. 4)}$$

The factor f_d controls the transition to the critical state. The factor f_s takes into account the increase of stiffness consecutive to an increase of the mean stress. These factors are function of the soil state defined by the void ratio e and the effective mean stress $P'(=(\sigma_r'+\sigma_\theta'+\sigma_z')/3)$ and of seven constitutive parameters (e_{c0}, e_{i0}, e_{d0}, α, β, hs and n). These parameters define the relationship between the minimum, critical and maximum void ratios and the means stress.

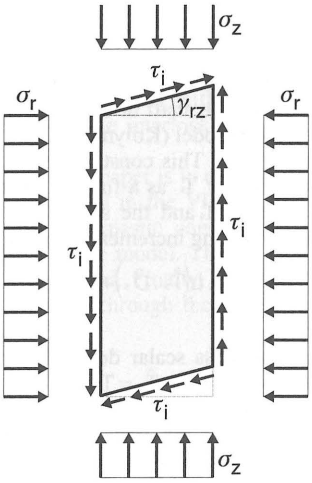


Fig. 4: Stress state assumed at the interface between 2 soil elements.

More detailed description of these parameters can be obtained in Bauer (1996), Gudehus (1996) or Vanden Berghe (2001).

4 CONSTITUTIVE LAW FOR TOE RESISTANCE

4.1 Assumptions

In order to calculate the toe resistance of the pile during the vibratory driving, the VIPERE model represents the soil under the pile by a cylinder (Fig. 5).

The section of the cylinder is equal to the section of the pile and its height is equal to 70% of the pile diameter. The soil at the pile base and the pile are supposed to stay permanently in contact. The model does not take into account the advent of gaps between the pile base and the soil as experimentally observed (Gudmani, 1997; Holeyman & al., 1998).

The determination of the height of the cylinder was suggested by the simplified semi-empirical equation calculating the immediate settlement of a foundation (Eq 5). This equation was developed based on the Boussinesq theory (Boussinesq – 1885), the theory of elasticity and experimental data. The influence coefficient I_p depends principally of the shape of the foundation (Steinbrenner – 1934). As a first approximation, the value of this coefficient can be taken equal to 0.7 for a solid vibratory driven pile.

$$s = q.B.I_P.\frac{1-\nu^2}{E} \qquad \text{(Eq. 5)}$$

Where s is the foundation settlement, q is uniform contact pressure, B is the foundation width, I_p is the influence coefficient, E is the soil stiffness and ν is the Poisson's ratio of soil

The behaviour of the soil cylinder at the pile toe is assumed to be hypoplastic and the element is supposed to be loaded in triaxial conditions (i.e. vertical stresses σ_z' and radial stresses σ_r' are principal).

As for the shaft resistance, the frequency of the cyclic loading is assumed to be high enough to consider that the excess pore pressure cannot be dissipated during the driving: the soil behaviour is assumed to be undrained ($\Delta e=0$). The pore pressure u is calculated based on the assumption that the total mean stress P ($=(\sigma_z+2.\sigma_r)/3$) of the considered soil element stays constant ($\Delta u=P'-P_0'$).

The resulting toe resistance F_{toe} is calculated based on the total axial stress acting on the pile base using equation 6:

$$F_{toe}=(\sigma_z'+u)A_{pile} \qquad \text{(Eq. 6)}$$

where σ_z' is the effective vertical stress, u is the pore pressure and A_{pile} is the pile section.

4.2 Hypoplastic model in triaxial shear

Similarly to the shaft resistance, the constitutive relationship used to calculate the stresses at the pile base is the hypoplastic model defined with Eq 3.

Based on the assumptions presented in the previous paragraph, the stress tensor and the strain rate tensor can be simplified as follow:

$$T_s = \begin{pmatrix} -\sigma_r' & 0 & 0 \\ 0 & -\sigma_r' & 0 \\ 0 & 0 & -\sigma_z' \end{pmatrix} \qquad \dot{D}_s = \begin{pmatrix} -\frac{1}{2}\Delta\varepsilon_z & 0 & 0 \\ 0 & -\frac{1}{2}\Delta\varepsilon_z & 0 \\ 0 & 0 & \Delta\varepsilon_z \end{pmatrix} \qquad \text{(Eq. 7)}$$

The vertical normal strain ε_z is calculated by dividing the pile displacement u_p by the height of the soil element considered under the pile ($=0.7.\varnothing_{pile}$).

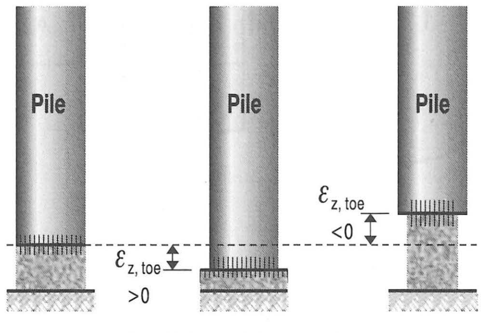


Fig. 5: Toe resistance model

Using these tensors, the stress rate at the pile base can be expressed as a function of the deviator q ($=\sigma_z'-\sigma_r'$) and the effective mean stress P' ($=(\sigma_z'+2.\sigma_r')/3$):

$$\Delta P' = f_s . |\Delta\varepsilon_1| . \frac{1}{3} . \left[\frac{q}{3.P'} . \mathrm{sign}(\Delta\varepsilon_1) + f_d . \sqrt{\frac{3}{2}} . a_1 \right]$$

$$\Delta q = f_s . |\Delta\varepsilon_1| . \left[\left(\frac{3}{2} . a_1^2 + \left(\frac{q}{3.P'} \right)^2 \right) . \mathrm{sign}(\Delta\varepsilon_1) + f_d . \sqrt{6} . a_1 . \frac{q}{3.P'} \right]$$

(Eq. 8)

The density factor f_d and stiffness factors f_s are functions of only the effective mean stress P' and the void ratio e.

5 MOTION EQUATION AND INTEGRATION

5.1 Pile Equilibrium

Fig. 6 shows the different forces acting on the pile. The vibrator induces a force resulting from the gravity force and the cyclic force induced by the vibration of eccentric masses. The soil reaction is divided into the resistance along the shaft and the toe resistance. The shaft and toe resistances are calculated using the hypoplastic model based on the relative displacement between the pile and the soil elements and the stress state in these elements.

The acceleration of the pile results from the unbalance between these forces (Eq. 9)

$$\ddot{u}_p(t) = \frac{M_{tot}.g + me.\omega^2.\sin(\omega.t) - 2.\pi.r_1.h_1.\tau_1(t) - F_{toe}}{M_{vib}}$$

(Eq. 9)

where $\ddot{u}_p(t)$ is the acceleration of the pile, M_{tot} is the total mass of the vibrator and the pile, M_{vib} is the vibrating mass consisting of the pile, the clamping device and the exciter block, me is the eccentric moment, ω is the angular frequency, r_1 is the equivalent radius of the pile, h_1 is the current pile penetration, τ_1 is the shear stress at the interface between the pile and the soil and F_{toe} is the base resistance

5.2 Soil Elements Equilibrium

Fig. 7 shows forces acting on each soil element modelling shaft resistance. The gravity force of the element is not taken into account. It is supposed to be balanced by the base resistance. The inter-ring reaction is calculated assuming an uniform distribution

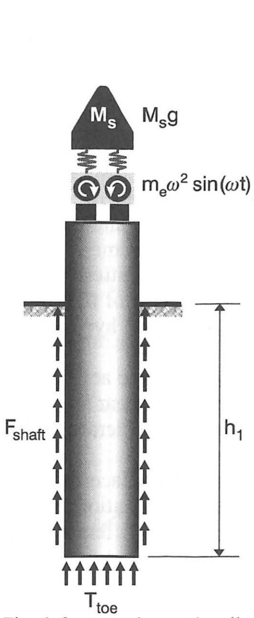


Fig. 6: forces acting on the pile

of the shear stress along the internal and external shaft of the soil element (Eq 10).

$$T_i = 2\,\pi\, r_i\, h_i\, \tau_i \qquad \text{(Eq. 10)}$$

where T_i is the inter-ring reaction between the elements i and i-1, r_i is the radius of the interface between elements i and i-1, h_i is the mean height of elements i and i-1, τ_i is the shear stress at the interface between elements i and i-1

Based on the inter-ring resistance, the acceleration $\ddot{u}(t)_i$ of each ring is calculated using equation 11. The displacement of the element is obtained by a double integration of this acceleration.

$$\ddot{u}(t)_i = \frac{(T_i - T_{i+1})}{M_i} \qquad \text{(Eq. 11)}$$

where $\ddot{u}(t)_i$ is the acceleration of the elements i, T_i is the inter ring reaction between the elements i and i-1, T_{i+1} is the inter ring reaction between the elements i+1 and i, M_i is the mass of the element i.

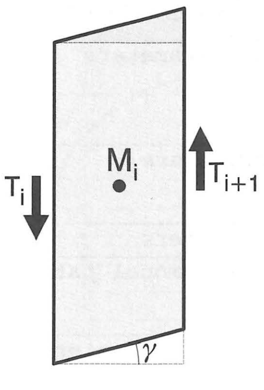


Fig. 7: forces acting on soil elements

The different steps in the calculation of the displacement of the soil elements i at the time step t+Δt can be summarised as follow:

- Based on the relative displacement between the considered element and its neighbours, calculation of the shear strain and the shear strain rate at each interface of the element;
- Based on the current stress state and the shear strain rate, calculation of the shear stress at each interface using the hypoplastic constitutive model;
- Calculation of the force acting an each interfaces by integration of the shear stress.
- Calculation of the acceleration of the considered element at the time t
- Calculation of the displacement of the element at the time t+Δt by integration of the motion equation using the central difference method.
- Return to the step 1

6 INTEGRATION PROCEDURE

Considering the motion equations of the pile and of each soil element, the system of Nr+1 motion equations of all the system can be developed. This system is not linear and cannot be solved by a direct inversion procedure. Indeed, the shear stress is calculated with the hypoplastic model that is expressed in an incremental shape. The model requires an explicit method to integrate the calculated acceleration.

The acceleration is evaluated as a function of the displacement using a central finite difference described as follow:

$$\ddot{u}_i(t)=\frac{u_i(t-\Delta t)-2.u_i(t)+u_i(t+\Delta t)}{\Delta t^2} \qquad \text{(Eq. 12)}$$

The value of the displacements at the time t+Δt is calculated using Eq 12 where $\ddot{u}_i(t)$ is evaluated with Eq 9 and 11.

7 MODEL RESULTS

This paragraph presents the results calculated by the model VIPERE. These results are based on the input parameters described in Table 1 and on the modelling of the pile penetrating a homogenous layer down to 12m depth.

7.1 Pile penetration

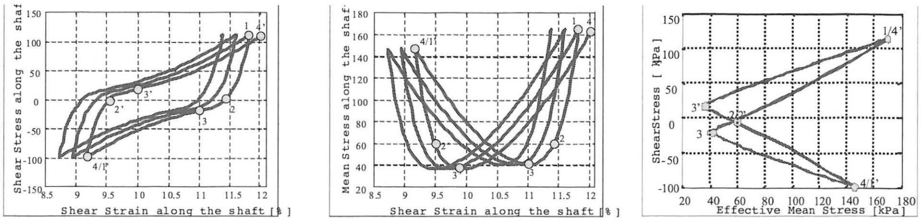
7.1.1 Shaft resistance

Along the shaft of the pile, the model VIPERE considers the soils condition is cylindrical simple sheared. The shear strain-shear stress (γ - τ) relationship is calculated using the hypoplastic constitutive law that takes into account the contractive and dilative behaviour of soil.

During the simulation of the vibratory driving, the τ_{rz} - γ_{rz} hysteresis loops look like bananas (Fig. 8-a) similar to the shape observed during typical laboratory testing. That result is the consequence of the 2 phases of dilation and 2 phases of contraction that are observed during each cycle (Fig. 8-b). When the shear strain rate changes sign, the soil behaviour becomes contractive and the shear decreases rapidly towards 0 (section 1-2 on Fig. 8). While the behaviour is contractive (section 2 –3), the shear stress stays low. However, when the soil skeleton is no more able to follow the imposed shear strain without trying to increase its volume (point 3), the behaviour becomes dilative and the resulting shear stress increases significantly (section 3-4) until the direction of the shearing is inversed. These phenomena are also illustrated by the butterfly shape characterising the stress path in the P'-q plane (Fig. 8-c).

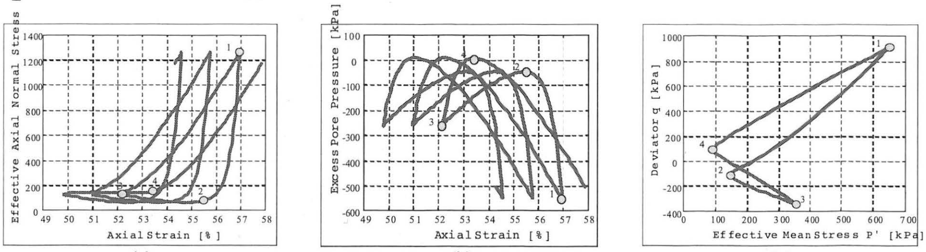
Table 1: Model parameters for simulations.

	Model Parameters		
Vibrator parameters	Eccentric Moment me = 46 kg.m Frequency = 33 Hz Vibrating Mass of Vibrator = 6700 kg Stationary mass of Vibrator = 3500 kg		
Pile Parameters	Pile Area = 167 cm² Pile Perimeter = 288 cm (diameter = 92cm) Pile Length = 12 m		
Soil Parameters	φ' = 30°	e = 0.60	
Hypoplastic model paramet	e_{c0} = 0.88 e_{d0} = 0.52 e_{i0} = 1.21	hs = 200 Mpa n = 0.35	α = 0.25 β = 1.10
Integration Parameters	R_{max} = 100 m Number of soil elements = 100 $\Delta t = 15.10^{-6}$ s		



(a) (b) (c)

Fig. 8: Soil resistance along the pile shaft during vibratory driving: (a) hysteresis loops, (b) mean stress evolution and (c) vertical shear stress vs effective radial normal stress



(a) (b) (c)

Fig. 9: Soil resistance at the pile base during vibratory driving: (a) evolution of the effective normal axial stress, (b) evolution of the pore pressure, (c) stress path.

7.1.2 Toe resistance

The VIPERE model calculates toe resistance based on the total axial normal stress applied at the base of the pile. The effective axial normal stress $\sigma'_{z\ toe}$ (Fig. 9-a) is evaluated with the hypoplastic model assuming the pile displacement solicits the soil under the toe in a triaxial way. The pore pressure u (Fig. 9-b) is deduced from the calculated effective mean stress P' assuming that the total mean stress P stays constant around the base of pile. During each cycle, 2 phases of contraction and 2 phases of dilation are observed. When the pile moves downwards, the effective normal stress increases dramatically during the dilation phase (section 4-1 on Fig. 9-a). When the direction of pile displacement changes (point 1), the effective axial normal stresses decreases rapidly to a low constant value (section 1-2) and stays around that value during the following contraction and dilatation phases (section 2-3-4). In fact, when the pile moves upward, the soil behaviour becomes active: it is the lateral stress that pushes the soil towards the pile base.

The evolution of the toe resistance calculated by the VIPERE model is quite similar to the model proposed by Dierssen (1994). When the pile moves downward, the toe resistance stays small during a part of the downward displacement whereas it increases rapidly when a certain threshold is reached. In the VIPERE model, the threshold is crossing from the contractive behaviour to the dilative behaviour. In the Dierssen model, when the pile moves upward, the toe resistance decreases rapidly and is equal to 0. In the VIPERE model, the toe resistance is not equal to 0 but is very small depending of the soil.

7.1.3 Penetration speed of the pile

The evolution of the penetration speed deduced from the net penetration of the pile is plotted on Fig. 10.

Fig. 11 presents the evolution during 3 cycles of the different forces acting on the pile. The active force of the vibrator is a sinusoid lightly shifted upwards to take the static weight into account. The resisting force is less regular due principally to the different phases of dilatancy and contraction of the soil. The curve of the resisting force looks symmetric because the pile cross sectional area chosen for this simulation is small comparing to the area of the pile shaft: the soil resistance is principally applied along the pile shaft and the toe resistance is very small (around 2.5% of the shaft resistance).

The acceleration resulting of the unbalance between acting and resisting forces is integrated (Fig. 12) to calculate the vertical velocity and displacement of the pile.

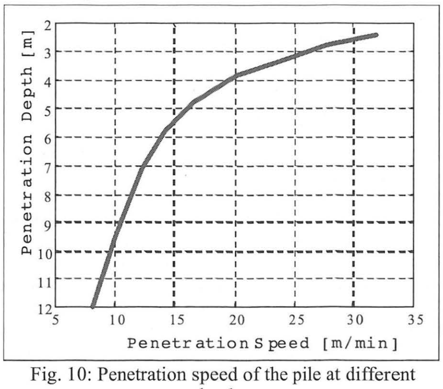


Fig. 10: Penetration speed of the pile at different depths.

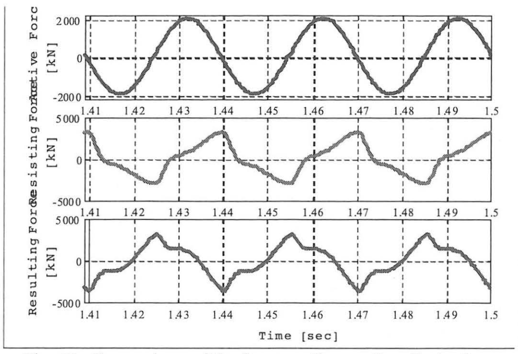


Fig. 11: Comparison of the forces acting on the pile (active force = $M_{tot}.g+me.\omega^2.\sin(\omega t)$; resisting force = $-(F_{shaft}+F_{toe})$)

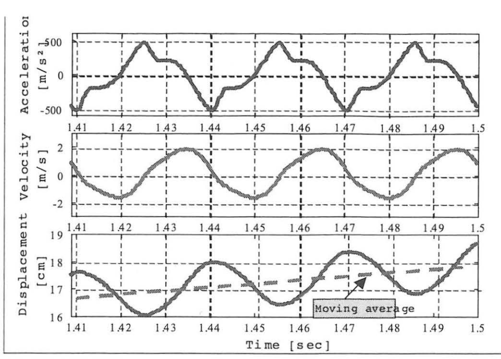


Fig. 12: Comparison of the acceleration, the velocity and the displacement of the pile

8 CONCLUSIONS

This paper has presented the VIPERE model calculating the penetration speed of a pile during its vibratory driving. More detailed description of this model can be found in Vanden Berghe (2001).

The model implements hypoplastic constitutive behaviour into a geometric model suggested by Holeyman (1994). The VIPERE model considers the pile as a rigid body and simulates the soil by a 1D radial discretisation. The soil behaviour is assumed to be hypoplastic and modelled using the Bauer (1996) and Gudehus (1996) constitutive law. The penetration speed are evaluated by integrating the equation of movement.

9 ACKNOLEGMENT

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