Hypoplastic modeling of vibratory pile driving

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Abstract: This paper describes a model able to simulate the penetration behaviour of a pile during its vibratory driving. The model actually implements hypoplastic constitutive behaviour into a geometric model suggested by Holeyman (1994). The computer code VIPERE, (for VIbratory PEnetration REsistance) considers the pile as a rigid body and simulates the soil around it using a 1D radial discretisation. The soil behaviour is assumed to be hypoplastic and modelled using the Bauer (1996) and Gudehus (1996) constitutive law. The penetration speed is evaluated by time integration of the equations of motion. The paper describes the model by specifying the assumptions relative to the pile and the soil behaviour, the equations used to evaluate the soil resistance along the pile shaft and at the pile base, and finally the procedure of integrating the motion equations. A typical simulation of pile driving is presented and discussed.

Keywords: pile, vibratory driving, undrained soil behaviour, cyclic loading, hypoplasticity.

1 INTRODUCTION

The VIPERE (for VIbratory PEnetration REsistance) model calculates the penetration speed of a pile at a given depth as a result of vibratory driving. The model considers the pile as a rigid body and represents the surrounding soil by discretising it into a series of concentric tubes (Figure 1). The interaction between the soil tubes is governed by a hypoplastic constitutive law.

To calculate the displacement of the pile as well as the wave propagation around the vibrating profile, the model follows a time-marching scheme to integrate the equations of motion. The acceleration of the pile in particular is calculated at each time step, based on the unbalance of the following forces acting on the pile:

- the static weight permanently acting of the pile, including the isolated "bias" weight (= $M_{total} \cdot g$),
- the vibrating force induced by the vibrator (= $me.\omega^2.\sin(\omega t)$ where me is the eccentric moment and ω is the angular frequency (= $2.\pi.N[rpm]/60$)),
- the friction resistance along the shaft of the pile F_{shaft}, and
- the toe resistance F_{toe}.

The forces acting on each discretized soil tube are, in addition to the inertial force, the friction (or shear) forces generated on the inside and outside walls as a result of the relative movement of the neighboring elements.

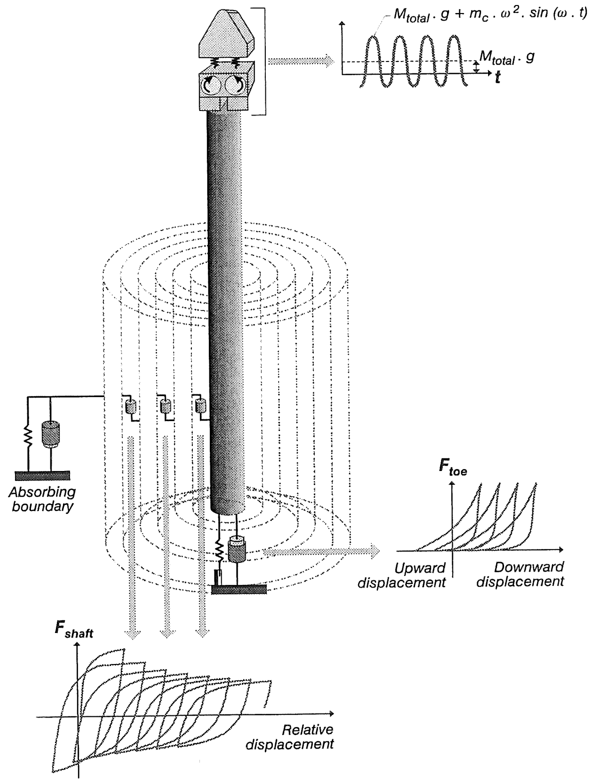


Figure 1 *Vibrodriving model (after Holeyman, 1994).*

The scope of this paper is to describe the main assumptions considered in the VIPERE model and to show how the hypoplastic constitutive relationships can be implemented into the model. After specifying the geometry of the model, some of the key features of hypoplasticity will be reviewed. Then two basic modes of deformation will be addressed: simple shear around the pile shaft to account for skin friction resistance, and triaxial compression to account for toe resistance at the pile base. The integration procedure is also discussed. Finally, the model performance is discussed through the analysis of simulation results.

The model borrows the geometric configuration of the HYPERVIB II model developed by Holeyman (1994, 1996) able to focus on the main component of the pile resistance during vibratory driving: the skin friction. The soil surrounding the pile shaft is lumped into a succession of tubes with depths h that slightly increase with the radial distance. That increase has been shown able to provide, in practical cases, an equivalent effect to the geometrical damping caused by the downward diffusion of waves beneath the level of the pile base, provided it is governed by:

$$\Delta h = \alpha . \Delta r \quad \text{(Eq. 1)}$$

where r is radial coordinate and α is related to the coefficient of dispersion (= 0.03 – Holeyman, 1996)

The weight of each soil tube is balanced by its base resistance. The radial discretisation is characterised by the number of rings (Nr) and the maximum radius (R_{max}) where an absorbing boundary is located. The stresses and strains are considered uniform with depth in each soil tube, allowing the currently selected model to be qualified as a 1-D radial model.

2 HYPOPLASTIC CONSTITUTIVE BEHAVIOR

Hypoplasticity is an alternative method to simulate the behaviour of materials that is neither linear elastic nor reversible. In contrast with the classic elastoplastic concept, the plastic strain rate is defined without explicit reference to any plastic potential function or yield surface. Those concepts are directly taken into account in the hypoplastic constitutive equation itself. The model does not, in its formulation, explicitly distinguish the elastic deformations from the plastic deformations. The material behaviour is described by a unique equation able to consider both loading and unloading (Kolymbas, 2000, Gudehus, 1996, Bauer, 1996).

This constitutive law evaluates the stress rate tensor $\dot{T}_s$ as a function of the current stress state tensor T_s and the strain rate tensor $\dot{D}_s$ based on the following incremental equation:

$$\dot{T}_s = f_s . \left[a_1^2 . \dot{D}_s + \hat{T}_s . tr(\hat{T}_s . \dot{D}_s) + f_d . a_1 . (\hat{T}_s + \hat{T}_s^*) . \|\dot{D}_s\| \right] \quad \text{(Eq. 2)}$$

$\hat{T}_s = T_s / tr(T_s)$ is the granular (or effective) stress ratio tensor, $\hat{T}^*_s = \hat{T}_s - \frac{1}{3} . \mathbf{1}$ is the deviatoric part of $\hat{T}_s$, f_s and f_d are scalars depending of the mean stress P' $(=(\sigma_r' + \sigma_\theta' + \sigma_z')/3)$ and the void ratio e. For the hypoplastic constitutive equation, the compressive stresses and shortening strains are defined negative in accordance to the convention in continuum mechanics. Only the scalar of the effective mean stress P' ($= -tr(T_s).\frac{1}{3}$) is defined positive in compression. a_1 is a scalar depending on the friction angle φ' of the soil and on the Lode angle, requiring the introduction of two parameters C_1 and C_2.

This rate type equation is able to consider an asymptotic behaviour in accordance to the critical state soil mechanics theory. The main advantage of such a framework is that it captures with a few parameters the vital ingredients of soil behaviour with respect to its interaction with a pile such as barotropy and pycnotropy, resulting in realistic description of coupling effects between spherical and deviatoric modes of deformation and variations in pore pressure.

The factor f_s takes into account the increase of stiffness consecutive to an increase of the mean stress. The factor f_d controls the transition to the critical state. These two factors are function of the soil state defined by the void ratio e and the effective mean stress P' and of seven constitutive parameters (e_{c0}, e_{i0}, e_{d0}, hs, α, β, n). These parameters define the relationship between the minimum, critical and maximum void ratios and the means stress. More detailed description of these parameters can be obtained in Bauer (1996), Gudehus (1996) or Vanden Berghe (2001).

On the basis of laboratory triaxial shear, direct simple shear, and oedometer tests, the seven hypoplastic parameters have determined by Vanden Berghe (2001) for Brusselian sand as follows:

e_{i0}, the maximum void ratio for a stress-free state = 1.21
e_{d0}, the minimum void ratio for a stress-free state = 0.52
e_{c0}, the critical void ratio for a stress-free state = 0.88
h_s, the granular hardness = 200 MPa
n, α, β, constitutive dimensionless constants equal to 0.35, 0.25, and 1.10, respectively.

In addition, C_1 and C_2 necessary to evaluate a_1 are determined from the friction angle (30° assessed) according to Herle's (2000) recommendations:

$$C_1 = \sqrt{\frac{3}{8}} . \frac{3 - \sin(\varphi')}{\sin(\varphi')} \qquad C_2 = \frac{3}{8} . \frac{3 + \sin(\varphi')}{\sin(\varphi')} \quad \text{(Eq. 3)}$$

3 CONSTITUTIVE LAW FOR SHAFT RESISTANCE

The uniform vertical shear stress τ_{rz} acting between two neighbouring tubes is calculated based on the relative displacements between soil tubes using the hypoplastic model (Gudehus, 1996; Bauer, 1996). The model assumes that each soil element deforms in simple shear condition as shown on Figure 2, similar to that enforced by a laboratory direct simple shear (DSS) test. The soil is assumed to be fully saturated and the frequency of the vibrator high enough to prevent any dissipation of the excess pore pressure during vibratory driving. Therefore, an undrained behaviour is assumed for the soil. Neither axial nor radial strains are permitted ($\varepsilon_r = \varepsilon_\theta = \varepsilon_z = 0$), leaving γ_{rz} as the only non-vanishing term of the strain tensor.

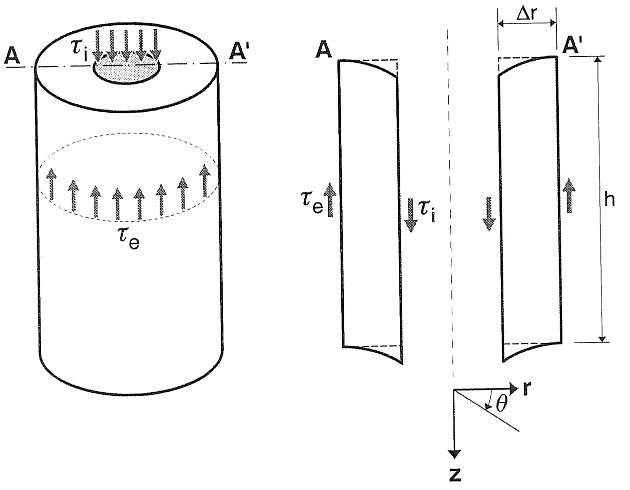


Figure 2 *Simple shearing in axisymetry*

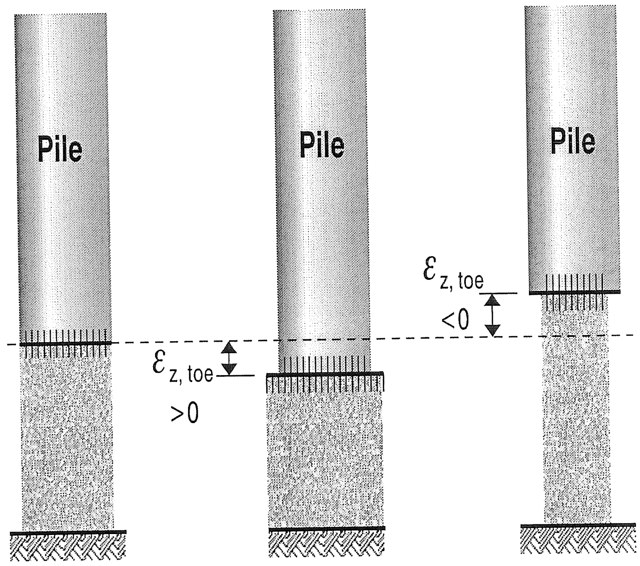


Figure 3 *Toe resistance model*

The shear strain γ_{rz}, calculated based on the relative displacements between soil tubes, enables one to evaluate the non-zero terms ($\tau_{r\theta} = \tau_{z\theta} = 0$) of the stress tensor by reducing equation 2 to :

$$\Delta\tau_{rz} = f_s \cdot \left|\Delta\gamma_{rz}\right| \cdot \left[\left(a_1^2 + \frac{2 \cdot \tau_{rz}^2}{(3P')^2} \cdot \right) sign\left(\Delta\gamma_{rz}\right) + f_d \cdot \sqrt{2} \cdot a_1 \cdot \frac{2 \cdot \tau_{rz}}{\cdot(3P')} \right]$$

$$\Delta\sigma_{r,\theta,z}' = f_s \cdot \left|\Delta\gamma_{rz}\right| \cdot \left[\frac{2 \cdot \tau_{rz} \cdot \sigma_{r,\theta,z}'}{(3P')^2} \cdot sign\left(\Delta\gamma_{rz}\right) + f_d \cdot \sqrt{2} \cdot a_1 \cdot \frac{\left(2.\sigma_{r,\theta,z}' - P'\right)}{3.(P')} \right]$$

(Eq. 4)

4 CONSTITUTIVE LAW FOR TOE RESISTANCE

In order to calculate the toe resistance of the pile during the vibratory driving, the VIPERE model represents the soil underneath the pile with a cylinder (Figure 3), similar to that loaded under a laboratory triaxial compression (TXC) test. The section of the cylinder is equal to the section of the pile base while its height is equal to 70% of the pile diameter, based on similitude and representative point considerations to match results of methods used in foundation engineering practice.

Under the current model development, the soil cylinder beneath the pile base is subjected to a uniform vertical strain ε_z governed by the pile base displacement; neither separation between pile base and soil, nor inertial effects are accounted for.

The hypoplastic behaviour of the soil cylinder beneath the pile toe will be developed for the triaxial conditions prevailing in the assumed model. σ_z' and σ_r' will the principal stresses and undrained conditions will be enforced ($\Delta e=0$). The resulting toe resistance F_{toe} is calculated based on the total axial stress acting on the pile base using equation 6:

$$F_{toe} = \left(\sigma_z' + u\right) A_{pile} \qquad \text{(Eq. 5)}$$

where σ_z' is the effective vertical stress, u is the pore pressure and A_{pile} is the pile section.

The pore pressure u is calculated assuming that the total mean stress P $=(\sigma_z+2.\sigma_r)/3$ of the considered soil element remains constant ($\Delta u = P' - P_0'$).

Based on these assumptions, Equation 2 can be expressed as a function of the deviator q ($=\sigma_z-\sigma_r$) and of the effective mean stress P' $=(\sigma'_z+2.\sigma'_r)/3$:

$$\Delta P' = f_s \cdot \left|\Delta\varepsilon_1\right| \cdot \frac{1}{3} \cdot \left[\frac{q}{3.P'} \cdot sign\left(\Delta\varepsilon_2\right) + f_d \cdot \sqrt{\frac{3}{2}} \cdot a_1 \right]$$

$$\Delta q = f_s \cdot \left|\Delta\varepsilon_1\right| \cdot \left[\left(\frac{3}{2} . a_1^2 + \left(\frac{q}{3.P'} \right)^2 \right) \cdot sign\left(\Delta\varepsilon_2\right) + f_d \cdot \sqrt{6} \cdot a_1 \cdot \frac{q}{3.P'} \right]$$

(Eq.6)

The density factor f_d and stiffness factors f_s are functions of only the effective mean stress P' and the void ratio e. Contrary to the simple shear case, the expression providing the value for a_1 can easily be simplified into:

$a_1 = 1/C_1$ if $q \geq 0$ (compression), and

$a_1 = 1/\left(C_1 + C_2 \sqrt{\frac{2}{3}\left|\frac{2q}{3.P'}\right|}\right)$ if $q \leq 0$ (extension).

5 MOTION EQUATION AND INTEGRATION

As explained in the introduction, the acceleration $\ddot{u}_p(t)$ of the pile is evaluated from :

$$\ddot{u}_p(t) = \frac{M_{tot} \cdot g + me. \, \omega^2 . \sin(\omega.t) - 2.\pi.r_1.h_1.\tau_1(t) - F_{toe}}{M_{vib}}$$

(Eq. 7), where

M_{tot} is the total mass of the vibrator and the pile,

M_{vib} is the vibrating mass consisting of the pile, the clamping device and the exciter block,

me is the eccentric moment,

ω is the angular frequency,

r_1 is the equivalent radius of the pile,

h_1 is the current pile penetration,
τ_1 is the shear stress at the interface between the pile and the soil and
F_{toe} is the base resistance
The inter-tube reaction is calculated by:

$$T_i = 2\pi r_i h_i \tau_i \qquad \text{(Eq. 8), where}$$

T_i is the shear reaction between elements i and i-1,
r_i is the radius of the interface between elements i and i-1,
h_i is the mean height of elements i and i-1,
τ_i is the shear stress between elements i and i-1
Based on the inter-tube shear resistance, the acceleration $\ddot{u}(t)_i$ of each soil tube is calculated using:

$$\ddot{u}(t)_i = \frac{(T_i - T_{i+1})}{M_i} \qquad \text{(Eq. 9), where}$$

T_{i+1} is the shear reaction between elements i+1 and i,
M_i is the mass of soil tube i.

Considering the motion equations of the pile and of each soil element, the system of Nr+1 motion equations of all the system can be developed. This system is not linear and cannot be solved by a direct inversion procedure. Indeed, the shear stress is calculated with the hypoplastic model that is expressed in an incremental shape. The model requires an explicit method to integrate the calculated acceleration.

The different steps in the calculation of the displacement of the soil elements i at the time step t+Δt can be sequenced as follows: (1) shear strain and shear strain rate, (2) shear force at each interface and axial reaction at pile base using the hypoplastic constitutive models, (3) acceleration of the lumped masses, (4) displacement of the element at the time t+Δt by integration of the motion equation using the central difference method, and (i+1) return to step (1).

6 MODEL RESULTS

6.1 Case Study

This section presents the results of a case study calculated by the VIPERE model. These results are based on the input parameters described in Table 1 and on the modelling of the pile penetrating a homogenous layer down to 12m depth. The results will be reviewed in terms of shaft and toe resistances for a reference penetration of 12 m. as well as in terms of pile penetration speed at that depth and overall penetration log.

6.2 Shaft Resistance

The model VIPERE can produce the shear stress-strain (τ-γ) relationships as calculated along the pile shaft using the hypoplastic constitutive law accounting for the contractive and dilative behaviour of soil. During the simulation of a few cycles of vibratory driving, the τ_{rz} - γ_{rz} hysteresis loops look like bananas (Fig. 4-a) similar to the shape observed during typical DSS laboratory testing.

Vibrator parameters	
Eccentric moment me = 46 kg.m	
Frequency = 33 Hz	
Vibrating mss of vibrator = 6700 kg	
Stationary mass of vibratory = 3500 kg	
Pile parameters	
Pile area = 167 cm^2	Pile length = 12 m
Pile perimeter = 288 cm (diameter = 92 cm)	
Soil parameters	
φ' = 30°	e = 0.60
Hypoplastic model parameters	
e_{c0} = 0.88	hs = 200 MPa
e_{d0} = 0.52	α = 0.25
e_{io} = 1.21	β = 1.10
n = 0.35	
Discretisation parameters	
R_{max} = 100 m	N_r = 100
$\Delta t = 15.10^{-6}$ s	

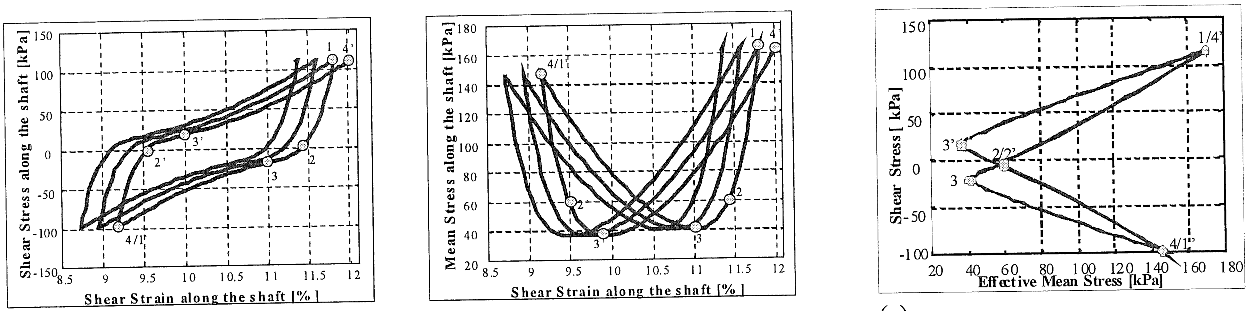
used
Table 1 *Model parameters for simulation~~s~~*
sandle

That result is the consequence of the two phases of dilation and two phases of contraction that are observed during each cycle (Fig. 4-b). When the shear strain rate changes sign, the soil behaviour becomes contractive and the shear stress decreases rapidly towards a nearly null value (path 1-2 on Fig. 4). While the behaviour is contractive (path 2 –3), the shear stress remains low. However, when the soil skeleton is no more able to follow the imposed shear strain without trying to increase its volume (point 3), the behaviour becomes dilative and the resulting shear stress increases significantly (path 3-4) until the direction of the shearing is reversed. These phenomena are also illustrated by the butterfly shape characterising the stress path in the P'-q plane (Fig. 4-c).

6.3 Toe Resistance

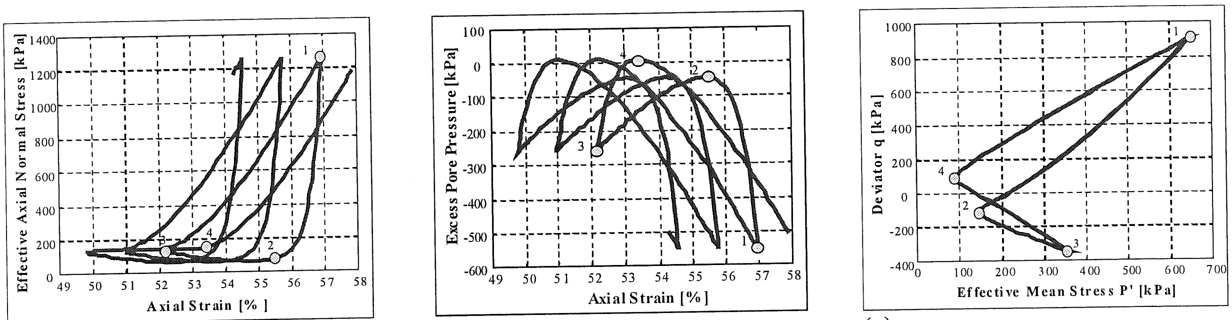
The VIPERE model calculates the toe resistance based on the total axial stress applied at the base of the pile. The effective axial normal stress $\sigma'_{z\ toe}$ (Figure 5-a) is evaluated with the hypoplastic model assuming an undrained behaviour of the soil cylinder underneath the pile toe under triaxial loading. The pore pressure u (Figure 5-b) is deduced from the calculated effective mean stress P' assuming that the total mean stress P remains constant around the pile base.

During each cycle, 2 phases of contraction and 2 phases of dilation are observed. When the pile moves downwards, the effective normal stress increases dramatically during the dilation phase (path 4-1 on Figure 5-a). When the direction of pile displacement reverses (point 1), the effective axial normal stresses decreases rapidly towards a low constant value (path 1-2) and remains around that value during the following contraction and dilatation phases (path 2-3-4). In fact, when the pile moves upward, the soil deforms under an ultimate active state with the radial stress pushing the soil towards the withdrawing pile base.



(a) (b) (c)

Figure 4 *Soil resistance along the pile shaft during vibratory driving: (a) τ-γ hysteresis loops, (b) mean stress evolution and (c) vertical shear stress vs. effective radial normal stress*



(a) (b) (c)

Figure 5 *Soil resistance at the pile base during vibratory driving: (a) evolution of the effective normal axial stress, (b) evolution of the pore pressure, (c) stress path*

The evolution of the toe resistance calculated by the VIPERE model is quite similar to the model proposed by Dierssen (1994). When the pile moves downward, the toe resistance stays small during a part of the downward displacement whereas it increases rapidly when a certain displacement threshold is reached. In the VIPERE model, the threshold corresponds to the switch from a contractive behaviour to a dilatant behaviour. In Dierssen's model, the toe resistance decreases rapidly to a null value when the pile moves upward. In the VIPERE model, the toe resistance is not nil but is indeed very small.

6.4 Pile Penetration

Figure 6 presents the evolution during 3 cycles of the different forces acting on the pile. The active force of the vibrator is a sinusoid lightly shifted upwards to take the static weight into account. The resisting force is less regular due principally to the different phases of dilation and contraction of the soil. The curve of the resisting force looks symmetric because the pile cross sectional area chosen for this simulation is small compared to that of the pile shaft: the soil resistance is principally applied along the pile shaft and the toe resistance is relatively small (around 2.5% of the shaft resistance).

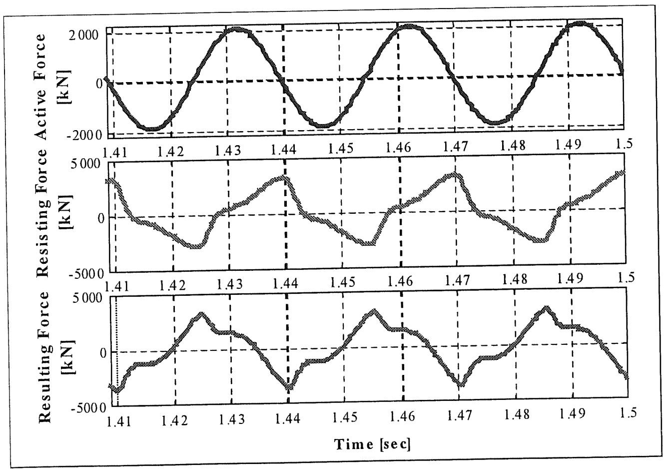


Figure 6 *Comparison of the forces acting on the pile (active force = $M_{tot}.g+me.\omega^2.sin(\omega t)$; resisting force = $-(F_{shaft}+F_{toe})$)*

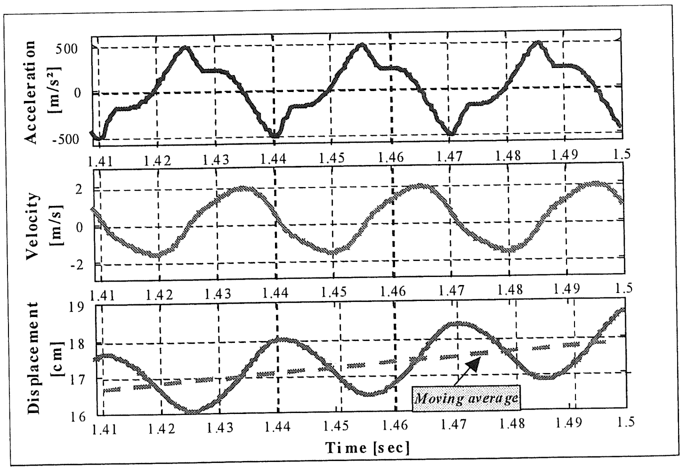


Figure 7 *Comparison of the acceleration, the velocity and the displacement of the pile*

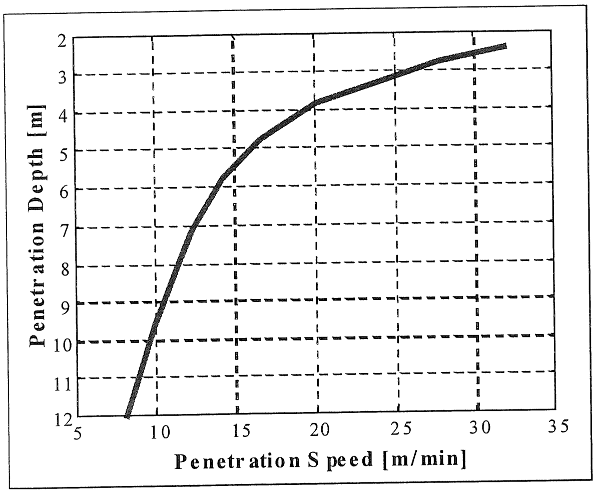


Figure 8 *Penetration speed of the pile versus depth*

The acceleration resulting of the unbalance between acting and resisting forces is integrated to calculate the vertical velocity and displacement of the pile, shown on Figure 7.

The evolution of the penetration speed deduced from the net penetration of the pile is plotted on Figure 8.

7 CONCLUSION AND FUTURE WORK

The VIPERE model has been at able to successfully implement hypoplastic constitutive behaviour into the geometric model suggested by Holeyman (1994). The model as highlighted the occurrence of two dilatant and two contractive phases around the pile shaft and beneath its base within each vibratory cycle. The calculated behaviour at the pile toe exhibits similarity with Dierssen's (1994) observations. The penetration speed can be realistically evaluated by the current model development.

Further refinement of the model should account for pore pressure cavitation cut-off, model multiple layers of soil elements and pile vertical compressibility, and possibly implement hypoplasticity into a 3-D finite elements package, able to handle dynamic effects.

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