Sheet pile vibro-driving: Assumptions vs. measurements

Whenham, V.
Belgian Building Research Institute, Brussels, Belgium
Holeyman, A.
Université Catholique de Louvain, Louvain-la-Neuve, Belgium

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ABSTRACT: Current models to assess the vibro-driveability of piles and sheet piles imply numerous restrictive assumptions relative to the influence of the power pack, the vibrator and the profile properties. These commonly accepted assumptions contrast with experimental evidence and can lead to significant consequences for the driveability prediction of the profile. The aim of this paper is to review measurements from recent full-scale test campaigns on instrumented sheet piles in order to illustrate and discuss the discrepancies between usual modeling assumptions and actual observations. The experimental results come from three test sites, located in Belgium (Limelette) and France (Merville, Montoir). The test sites are characterized respectively by loam, tertiary clay and sand soil conditions.

1 INTRODUCTION

There are three main methods to install piles or sheet-piles in the soil: impact driving, jacking and vibrating. Because of its low cost, high production and limited environmental disturbances, the last technique is the most commonly used technique in ‘‘relatively soft’’ soils. The vibratory driving technique can also be used for extracting sheet piles. Its major limitation is a lack of guidelines in relation to driving refusal. Impact driving can generate high energy under difficult soil conditions, but generates noise and high vibrations in the soil. Jacking is expensive and is used only where sensitive environmental conditions are encountered.

Vibratory driving is based on degradation and liquefaction of the soil around a vibrated profile. A vibrator, clamped on top of the pile or sheet pile, induces to the soil a cyclic load which leads to the degradation of soil strength and the build-up of the pore pressure. Due to this significant reduction of soil resistance, the pile can be sinked into the soil under gravity forces. The efficiency of the technique depends upon numerous parameters such as the pile to be driven, the selected vibrating equipment and the encountered soil conditions.

Over the years different engineering design tools and more fundamental modeling approaches have been suggested to assess the driveability of piles and sheet piles. These methods vary widely in the way they account for mechanical engineering principles and in the way the soil behaviour is modeled. Because of the difficulty to accurately represent the mechanisms at play, these methods make drastic assumptions with respect to the vibrator-pile-soil interactions. These assumptions need to be verified on the basis of experimental observations, by preference by means of fully instrumented real scale driving tests. Full-scale testing provides the valuable advantage be free from improper boundary conditions and/or scale inconsistencies.

In this paper, common assumptions on the vibro-driving process are firstly reviewed. Then experimental results from recent full scale tests are presented. Finally discrepancies between usual modeling assumptions and actual observations are discussed.

2 COMMON ASSUMPTIONS FOR ASSESSING VIBRO-DRIVEABILITY

The theory underlying the vibratory driving technique is reviewed with emphasis on common assumptions adopted in vibro-driveability prediction methods.

2.1 Vibrator action

The mechanical action of a vibrator onto a profile consists of two parts: a vibratory action produced by counter-rotating eccentric masses actuated within the vibrating part of the vibrator (exciter block), and a stationary action induced from gravity forces.

The exciter block is connected to the profile via a clamping device and is suspended to a carrier. The suspension device includes a vibration isolator mechanism consisting of a quasi-stationary mass

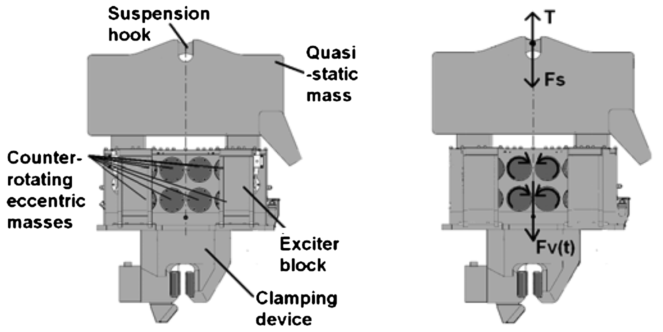

Figure 1. Example of vibrator (ICE 36RF-ts).

directly suspended to the suspension hook and an intervening spring, generally consisting of elastomer pads. The net quasi-stationary action on soil is the weight of the pile mass, vibrator mass and clamping device deduced by the suspension force exerted by the crane operator.

$$F_s = (M_{vib,tot} + M_{cl} + M_p) \cdot g - T \tag{1}$$

where

F_s = quasi-stationary force exerted on the soil [N]
$M_{vib,tot}$ = total mass of the vibrator [kg]
M_{cl} = mass of the clamp [kg]
M_p = mass of the profile [kg]
T = suspension force exerted by the crane (generally omitted in the prediction methods) [N].

The amplitude of the vibratory action resulting from the centrifugal forces of the symmetrically moving eccentric masses is given by

$$F_v(t) = me.\omega^2 \sin(\omega t) = F_c \sin(\omega t) \tag{2}$$

where

F_c = max. centrifugal force of the vibrator [N]
me = eccentric moment of the vibrator [kg.m]
ω = angular frequency of the vibrator [rad/s]

In this relationship the maximal centrifugal force of the vibrator F_c takes a constant value: it is assumed that the angular frequency of the vibrator and its eccentric moment remain equal to their nominal value during the whole vibro-driving phase. Provided the center of gravity of the rotating masses belongs at all times to the profile neutral axis, the exciter block is supposed to exert a purely longitudinal force onto the profile. Therefore transversal and flexural effects are generally ignored in analyses. Under the additional assumption that the pile behaves as a rigid body rigidly connected to the exciter block and neglecting the movement of the quasi-stationary mass, the vibrating force leads to a displacement with amplitude of

$$d_0 = me/M_{dyn} \tag{3}$$

where M_{dyn} = dynamical mass (vibrating part of the vibrator, clamp and pile) [kg]

2.2 *Assumptions on the (sheet) pile behaviour*

Steel and concrete profiles can be installed by vibro-driving. They are generally characterized by the following properties:

A = profile cross section [m]
L = profile length [m]
χ = circumference of pile [m]
M_p = mass of the pile [kg]

When using these parameters, it is considered that a sheet pile has the same behaviour as an equivalent cylindrical or prismatic pile, neglecting the transversal and flexural properties of the profile as well as the influence of the profile shape. Moreover, most of the methods omit the influence of the elasticity parameters, assuming that the profile behaves as a rigid body.

2.3 *Assumptions on the vibrator-(sheet) pile connection*

The vibrator is assumed to be rigidly clamped to the sheet pile. It is considered that the pile has the same movement as the vibrating part of the vibrator and that no bending moments are transmitted.

2.4 *Assumptions on the (sheet) pile-soil interaction*

The understanding of the pile-soil interaction during the vibro-driving process is of prime importance. According to Holeyman (2000), as the profile undergoes a vibratory vertical motion, it communicates to the lateral neighboring soil shear stresses and shear strains, and it forces normal and potentially convective movement of soil below the pile toe. Therefore the understanding of the shear stress/shear strain relationship within the soil becomes of paramount importance. The constitutive relationships that represent the complex large-strain, dynamic and cyclic shear stress-strain strength behaviour of the medium surrounding the vibrating profile require the characterization of the following elements:

– Static stress-strain law expressing nonlinear behaviour under monotonic loading and hysteresis upon strain reversal;
– Shear modulus at small strains and ultimate shear strength;
– Softening and increase of hysteretic damping with increasing strain;
– Effect of strain rate on initial shear modulus and ultimate strength;
– Degradation of properties resulting from the application of numerous cycles;
– Generation of excess pore pressure leading substantial loss of resistance and possibly to liquefaction.

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Different constitutive laws have been proposed in the literature to deal with these requirements, see a.o. Holeyman (1994, 1996, 2000), Cudmani (2001), Rausche (2002), Vanden Berghe and Holeyman (2002), Cudmani and Sturm (2006) and Sieffert (2006).

2.5 *Conclusions*

Common assumptions for assessing vibro-driveability can be summarized as follows:

- The net quasi-stationary action on soil is given by equation (1); the suspension force (T) is generally omitted.
- The amplitude of the vibratory action is given by equation (2). The maximal centrifugal force F_c is assumed to be purely longitudinal and constant during the whole driving phase.
- It is most often considered that a sheet pile has the same behaviour as an equivalent cylindrical or prismatic pile, neglecting the transversal and flexural properties of the profile as well as the influence of the profile shape.

Besides, none of the currently available models can reproduce all of the aspects of real sheet pile-soil interaction behaviour. Therefore, assumptions related to the soil-pile interactions vary significantly from one method to the other.

3 METHODS FOR ASSESSING SHEET PILE VIBRO-DRIVEABILITY

Based on a combination of the above theory and experimental investigations, different models have been developed over the years for assessing the vibro-driveability of profiles.

3.1 *Force equilibrium models*

Force equilibrium models are the simplest design tools to predict which vibrator is necessary to install a sheet pile without problems.

As an example, Van Baars (2004) has recently proposed the following equation:

$$F_c \geq \gamma\left(LA\beta \exp\left(\frac{q_{c,tip}}{q_{c,ref}}\right) + L\chi\alpha q_{c,tip}\right) \tag{4}$$

In which

$\gamma = 1.20$ [-], $\alpha = 0.001$ [-],
$\beta = 220$ [kN/m], $q_{c,ref} = 8.7$ [MPa]
$q_{c,tip}$ = cone penetration resistance at tip of pile [kPa]

This equation has been calibrated on calculations made for several standard cases (Azzouzi, 2003) with the computer model HIPERVIB-I developed by Holeyman (1993). The method has been further verified on the basis of 18 field tests with varying sheet piles, vibrators and soil conditions (Van Baars, 2004).

3.2 *Integration of law of motions*

Comprehensive accounting of the laws of mechanics requires that movement be described at all times from inertial equilibrium conditions. The simplest models involve a single degree of freedom. These models assume that the pile behaves as a rigid body moving only vertically. Newton's law can therefore be applied to the dynamic mass:

$$a = \frac{me\omega^2 \sin(\omega t)}{M_{dyn}} \tag{5}$$

Holeyman (1993) has suggested a method that integrates the inertial effects of the excess force, defined as the difference between the sinusoidal driving force and the opposing soil resistance. The soil degraded resistance at the toe and along the shaft is estimated from CPT results where the friction ratio and acceleration ratio are used to assess the severity of degradation. The method has been verified and liquefaction parameters further refined through calibration with full-scale tests (BBRI, 1994).

Other authors (Dierssen 1994, Gonin 1998, Sieffert, 2006) have proposed similar methods based on a single degree of freedom system. The main limitation of these studies is a lack of validation and guidelines in relation to soil parameters.

Further studies have lead to the development of radial (Holeyman, 1993) and longitudinal (Gardner 1981, Chua et al. 1981, Moulai-Khatir 1994, GRL 1998) 1D models.

3.3 *Finite element models*

A few researchers have used the finite element method to simulate pile vibrodriving (Chow and Smith 1984, Smith and To 1988, Leonards et al. 1995, Grabe et al. 2006, Cudmani and Sturm 2006, Mahutka and Grabe 2006). As an example, Cudmani and Sturm (2006) have compared model tests in calibration chambers and numerical simulations with the FEM to investigate the mechanisms of the tip resistance during static, alternating and dynamic penetration in granular and saturated soft soils. According to the authors, the good accordance between experimental and numerical results evidences the ability of the proposed numerical models to predict the tip resistance (except for cases where grain crushing becomes important).

3.4 *Conclusions*

Models vary in complexity in function of the way they account for mechanical engineering principles and in the way the soil behaviour is modelled. To the authors' knowledge, the assumptions summarized in section 2.5 are however adopted in all the methods currently available for assessing sheet pile vibro-driveability.

4 FULL SCALE TESTS

Because the processes underlying the vibratory technique are far from being fully understood, full scale sheet pile driving tests are necessary to verify and calibrate the theoretical models, as well as to improve our understanding of the phenomena at play. During full-scale driving tests, parameters such as the frequency, load, energy, penetration velocity, and soil particles velocity are typically monitored.

The experimental results presented below come from three test sites, located in Belgium (Limelette) and France (Merville, Montoir).

The tests from Merville and Montoir have been conducted within the framework of a research project ''Vibrodriving'' organized in France between November 2000 and December 2005. It was an operation of the Network for Civil and Urban Engineering (RGCU or IREX in French) of the Ministries for Public Works and for Research. LCPC and INSA of Strasbourg have carried out most of the data processing related to the experiments of that project.

4.1 Merville

At Merville an open tube and a double sheet pile (a.o.) were driven in Flanders clay. General information about the tests is summarized in Table 1. Geotechnical parameters of the test site are presented in Table 2.

4.2 Montoir

At Montoir a closed-end tube (with a length of 32 meters) was driven in a sandy-clay soil down to a depth of 18 meters. General information about the tests is summarized in Table 3. Geotechnical parameters of the test site are presented in Table 4.

4.3 Test site of Limelette (2003–2007)

A series of tests have been conducted on the test site of Limelette (Belgium) between 2003 and 2007, where instrumented sheet piles have been installed and continuously monitored. The parameters for two of these tests are described in Tables 5 and 6. The soil conditions at the test site consist of a medium to stiff silty layer underlaid by compact sand (see Fig. 2). The groundwater table lies approximately 60 m below ground level.

Table 1. Profile tested and nominal vibratory parameters at Merville (Arnould et al., 2005)

Site	Merville	
Profile	Open Tube	Sheet pile
Perimeter [cm]	160	382
Section [cm²]	266	247
Mass of the profile [kg]	2518	2455
Length [m]	12.3	13
Nominal me [kg.m]	46	46
Nominal Frequency [Hz]	26	26
Dynamic mass (excl. pile) [kg]	4910	5660
Total mass [kN]	75.5	111.4

Table 2. Geotechnical characterization – Merville (Arnould et al., 2005)

Depth [m]	Nature	P_l^* [MPa]	E_M [MPa]	q_c [MPa]
0 to 2.2m (at 1m)	Loam	0.25	3.7	0.7
2.2 to 42m (at 4m)	Flanders	0.75	14	2
(at 16m)	clay	1.8	35	5

P_1^* = Menard limit pressure, E_M = Menard E-modulus, q_c = cone resistance

Table 3. Profile tested and nominal vibratory parameters at Montoir (Arnould et al., 2005)

Site	Montoir
Profile	Closed tube
Perimeter [cm]	106
Section [cm²]	191
Mass of the tube [kg]	4057
Length [m]	32
Nominal me [kg.m]	46
Nominal Frequency [Hz]	26
Dynamic mass (excl. pile) [kg]	5250
Total mass [kN]	118.8

Table 4. Geotechnical characterization – Montoir (Sieffert & Borel, 2004)

Depth [m]	Nature	P_l^* [MPa]	E_M [MPa]	SPT [N]	q_c [MPa]
0-4.2m	Sand	1	10	18	2 to 16
4.5-8.5m	Sand	1	7	7 to 17	10 to 18
8.5-13m	Slightly clayey fine sand	0.6	5	7 to 16	3 (peak 10)
13-22.5m	Sand with clayey lens	0.7	5	1 to 10	1.75 (peak 8 to 12)

Table 5. Sheet piles tested at Limelette

Site	Limelette	
Profile	Sheet pile no 1 (2004)	Sheet pile no 2 (2007)
Perimeter [cm]	414	330
Section [cm²]	311	199
Mass [kg]	4900	3300
Length [m]	20	20

Table 6. Nominal vibratory parameters – Limelette

Site	Limelette	
	Sheet pile no 1 (2004)	Sheet pile no 2 (2007)
Eccentric moment [kg.m]	35 (nominal value)	26 (deduced from meas.)
Nominal Frequency [Hz]	33	38
Dynamic mass [kg]	10000	9800
Total mass [kN]	138	116

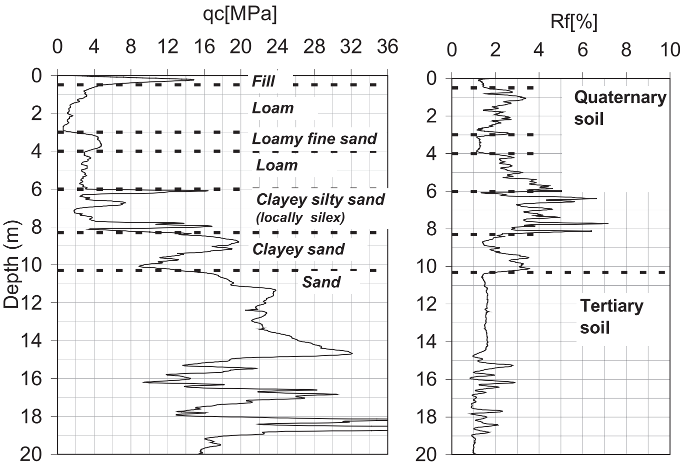

Figure 2. CPT-E results – Test site of Limelette.

5 ANALYSIS OF EXPERIMENTAL RESULTS

5.1 *Vibrator action*

Full-scale driving tests allow monitoring field parameters related to the vibrator action, essentially the vibratory frequency and amplitude generated on top of the sheet pile as well as the transmitted loading and energy. These field-related parameters can be compared to the theoretical vibratory parameters.

Acceleration measurements performed on top of the sheet pile can be used to evaluate the dominant frequency and the displacement amplitude of the profile, assuming that the mean acceleration is nihil on the average and that velocity is constant within a period. The dominant driving frequencies calculated by double integration of acceleration measurements and expressed as a function of the penetration depth are represented in Fig. 3a and 3b respectively for tests conducted at Merville and Limelette. The results from Merville show a significant decrease in frequency with penetration depth, while the frequencies at Limelette are almost constant during all the vibratory phase, exhibiting only a slight tendency to decrease with depth. These observations can be explained by the interactions between the vibrator parameters and the power pack, as explained in Holeyman & Whenham (2008).

Fig. 4 presents the evolution of the displacement amplitude on top of the profiles in function of the penetration depth, for the tests performed at Merville and Limelette. As observed with the frequency, the displacement amplitude is decreasing rapidly with depth for the test conducted at Merville, while the displacement amplitude is almost constant for the test conducted at Limelette. The decrease in displacement amplitude at Merville can be explained by the friction provided by the clay, which increases with the penetration depth of the profile. Because the displacement amplitude is derived from a double integration of the acceleration measurements, this decrease also reflects the otherwise observed decrease in dominant frequency.

The actual load transmitted to the sheet pile can be derived from strain gauges transducers measurements performed on the top of the sheet pile. The alternating part of the measurements can be compared with the vibratory force theoretically transmitted to the top of the sheet pile. Taking the example of the tests performed at the test site of Limelette (sheet pile no 1, 2004), the following assumptions can be made in a first approximation:

- The driving circular frequency ω(t) and the eccentric moment me(t) are constant during the complete vibratory stage (Figs. 3b & 4b).
- The system is free (the soil influence is not considered);
- Losses in the connections between the vibrator and the sheet pile are neglected, as well as the losses due to bending and twisting of the sheet pile (transmitted forces are assumed to be purely vertical).

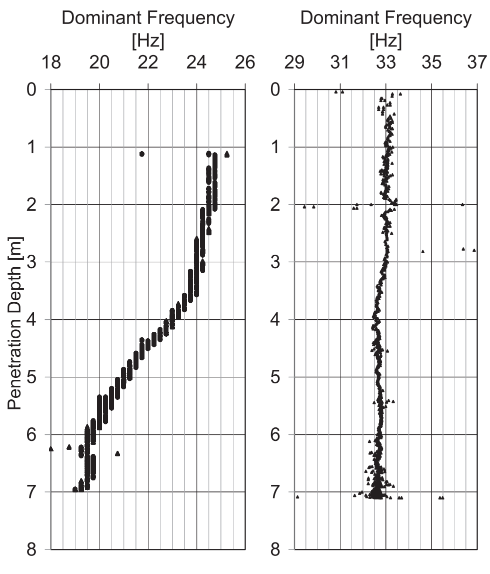

Figure 3. Evolution of the dominant frequency in function of the penetration depth: (a) Sheet pile tested at Merville, (b) Sheet pile no 1 tested at Limelette 2004.

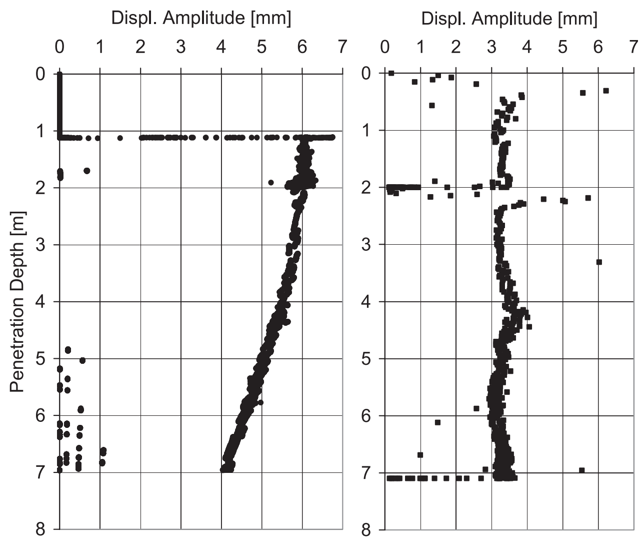

Figure 4. Evolution of the vibratory amplitude in function of the penetration depth: (a) Top of sheet pile tested at Merville, (b) Top of sheet pile no 1 tested at Limelette 2004.

Based on these assumptions, the theoretical amplitude of the vibratory force $F_{v,theo}$ can be expressed according to equation (6) (Arnould et al., 2005). The measured vibratory part of the driving force $F_{v,mes}$ is given in Fig. 5. It can be observed that the measured force amplitude is approximately 2/3 of the theoretical value.

$$F_{v,theo} = m_e \cdot \omega^2 \frac{1}{1 + \frac{m_{vib}}{m_{pile}} \cdot \frac{\alpha}{tg(\alpha)}} \approx 840\,\text{kN} \qquad (6)$$

with $\alpha = \frac{\omega \cdot L}{c} \approx 0.83$

L = length of the sheet pile [m];
c = wave velocity [m/s]
m_{vib} = total vibratory mass (excluding the pile) estimated to 5100 kg
m_{pile} = mass of the sheet pile estimated to 4900 kg

The energy transmitted to the top of the sheet pile is given by equation 7, where Force(t) is the total force exerted on the top of the sheet pile (derived from strain signals) and Velocity(t) is the velocity measured at the top of the sheet pile (derived from acceleration signals).

$$E_{enthalp} = \int (Force(t)) \cdot (velocity(t))dt \qquad (7)$$

The phase difference between velocity and force deduced from the acceleration and strain signals is shown in Fig. 6a. From this result, the power transmitted to the top of the sheet pile can be calculated, as presented in Fig. 6b.

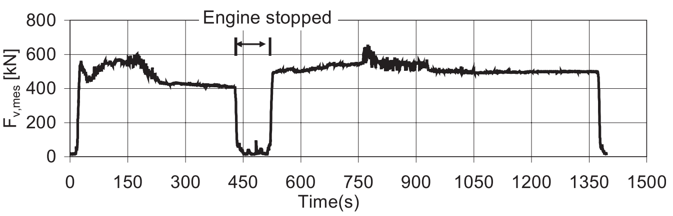

Figure 5. Measured vibratory part of the driving force – Limelette 2004.

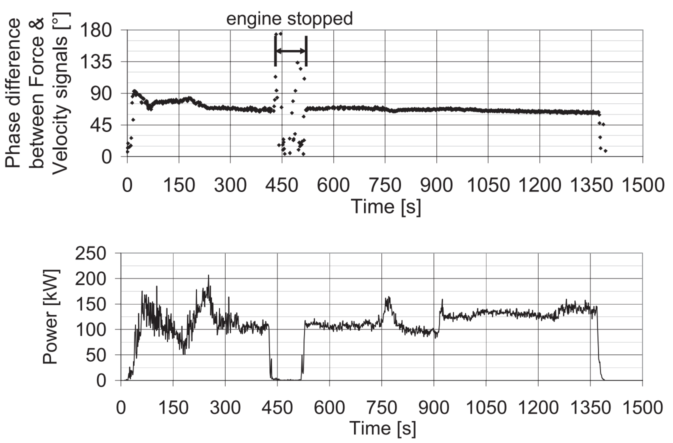

Figure 6. Phase difference between velocity and force & calculated transmitted power – sheet pile no 1, Limelette (2004).

5.2 *Observed (sheet) pile behaviour*

From accelerometers positioned in the three directions and at different locations on the sheet pile, it is possible to investigate the actual movements of the sheet pile in comparison with the usual assumptions adopted in modeling approaches.

Results of displacement amplitudes interpreted from measurements for the tube tested at Merville and for the sheet pile (no 2) tested at Limelette are presented in Fig. 7. For the tube tested at Merville, the displacement amplitude at the base is lower than the displacement amplitude at the top, while the inverse is observed for the sheet pile tested at Limelette. These observations can be explained considering steady state solutions for the movement of an elastic body subjected to a vibratory force $F_v(t)$. For free and fixed bottom boundary conditions, these solutions are given respectively by equations (8) and (9) (Arnould et al., 2006). A more realistic boundary condition is obtained by replacing the soil resistance by a spring of stiffness K_T, as illustrated in Fig. 8. In

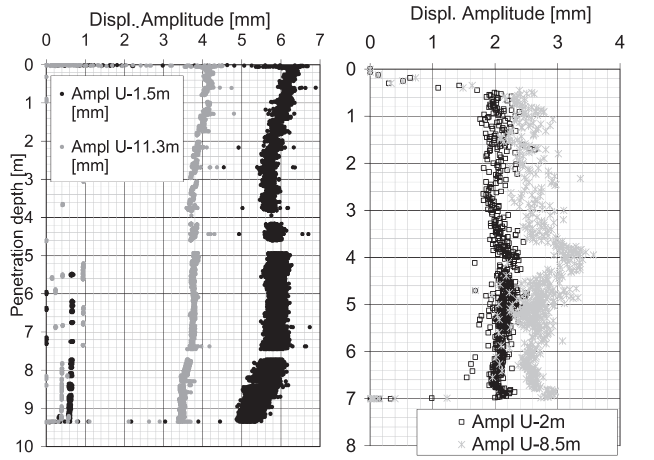

Figure 7. Displacement amplitude for (a) the tube tested at Merville (position of the measurements: 1.5 m and 11.3 m from the top), (b) the sheet pile no 2 tested at Limelette (2007) (position of the measurements: 2 m and 8.5 m from the top).

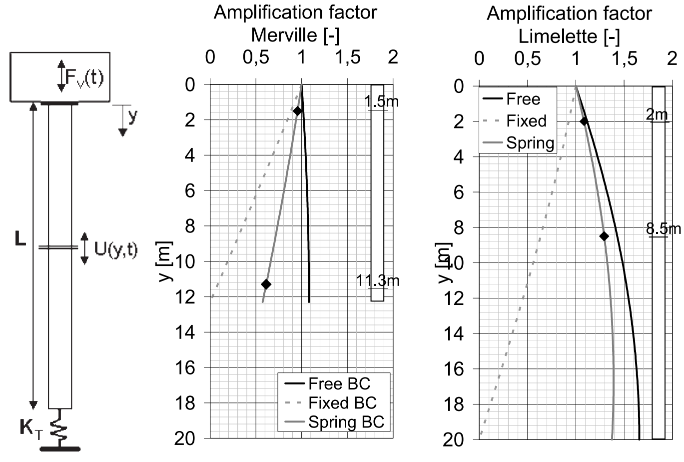

Figure 8. Amplification factors of the displacement amplitudes, deduced from the steady state solutions and expressed as a function of the position of measurement on the profile.

this case the displacement amplitude is given by equation (10).

$$u(y,t) = \frac{\cos k(L-y)}{\cos kL} \cdot u(0,t) \tag{8}$$

$$u(y,t) = \frac{\sin k(L-y)}{\sin kL} \cdot u(0,t) \tag{9}$$

$$u(y,t) = \frac{\left(\frac{Z_c.\omega}{K_T}\right).\cos k(L-y) + \sin k(L-y)}{\left(\frac{Z_c.\omega}{K_T}\right).\cos(kL) + \sin(kL)}.u(0,t) \tag{10}$$

where

u(y,t): displacement amplitude [m]
ω = driving circular frequency [rad/s]
L = length of the sheet pile [m];
c = wave velocity [m/s]
Z_c = profile impedance [N.s/m]
K_T = stiffness of the spring [N/m]

In Figs. 9 and 10, the transverse (horizontal) movements of a vibrated sheet pile are illustrated on the basis of measurements performed on a sheet pile tested at Limelette. These figures also show the influence of the used clamping devices.

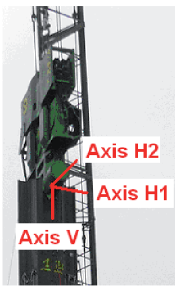

Figure 9. Definition of axes H1, H2 and V.

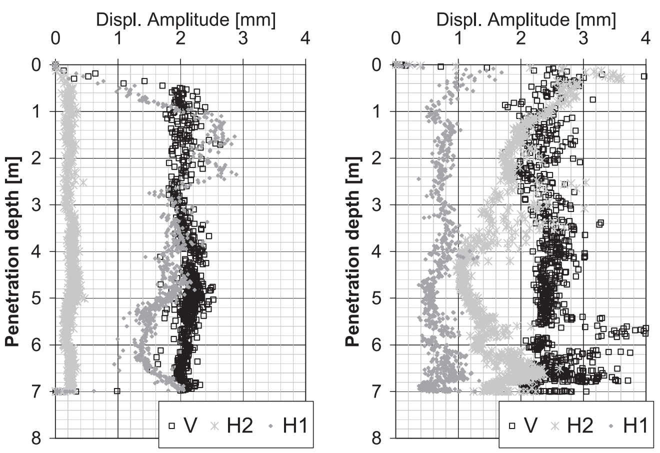

Figure 10. Lateral movements of sheet piles vibro-driven at the Limelette test site, (a) with one clamp positioned on the flange of the sheet pile, (b) with two clamps positioned on the webs of the sheet pile – Sheet pile no 2, Limelette (2007).

5.3 *On the (sheet) pile-soil interaction*

A last parameter that can be investigated with full scale driving test is related to the (sheet) pile-soil interaction. As an example, Fig. 11 shows the detail of an acceleration measurement obtained at the test site of Montoir. The harmonics embedded in the signal are induced by the high base resistance encountered at Montoir. Measurements presented in Fig. 11 clearly differ both in amplitude and frequency content from the theoretical sinusoidal accelerations that should result from equation (2) (under the assumption that the pile behaves as a rigid body). Because the soil behaviour is dependent on both the frequency and amplitude of the solicitation, it is very likely that its driving resistance will be influenced by these discrepancies between theoretical and observed accelerations.

5.4 *Conclusions*

The main discrepancies between commonly adopted assumptions and observations from full scale driving tests are the following:

- Amplitude and frequency parameters are not constant during the driving phase.
- Force and power transmitted to the sheet pile may reach only 1/2 to 2/3 of the nominal values.
- The position of the clamping device on the profile has an influence on the movement actually enforced to the top of the profile.

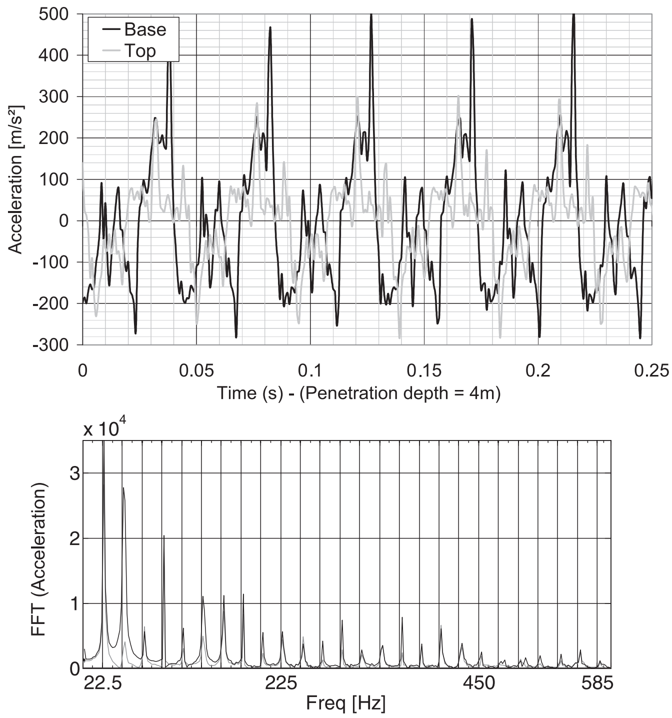

Figure 11. Detail of an acceleration signal obtained at Montoir – (a) time record, (b) frequency analysis.

– The effect of the sheet pile lateral flexibility on its penetrative behaviour is clearly evidenced when measuring the transverse movements of the sheet pile.
– The shape of the signals can significantly differ both in frequency and amplitude from the theoretical sinusoid.

6 CONCLUSIONS

Current models for vibro-driving of (sheet) piles imply numerous restrictive assumptions relative to the influence of the power pack, the vibrator and the (sheet) pile properties. First, the power pack-vibrator interactions are most often ignored. That leads to the assumption that the vibrator parameters remain constant during the driving process. Secondly, the sheet pile is generally represented by a rigid body, without consideration for either its longitudinal or lateral flexibility, and without consideration for phenomena resulting from the potential vibrator-pile-soil interactions. The above commonly accepted assumptions contrast with experimental evidence and can lead to significant consequences for the driveability prediction of the profile.

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