SPHERE PENETRATION EXPERIMENTS IN VERTICALLY VIBRATED SAND

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ABSTRACT

The paper concerns vibrodriving and vibrocompaction processes applied to granular soils and presents experiments able to characterize the behaviour of sand subjected to vertical vibration. Soil degradation phenomena arising during vibro-driving are first described and the vibro-fluidization of dry granular matter is discussed. The paper concentrates on the shear strength degradation of vibrated dry granular soils. Vertical vibration is then particularly investigated. The volume change during vertical vibration is described and the sand behaviour during vibration is also discussed. One focuses on the influence of the acceleration on the dry density after vibration. Finally, new sphere penetration experiments (SPE) based on the conceptual work of Barkan (1962) are performed.

When sand is vertically vibrated under gravitational field, three types of behaviour are identified, depending on the acceleration amplitude : the densification behaviour, the surface instability behaviour, and the vibro-fluid behaviour. The different patterns of particles recirculation are described. The boundaries between the different regimes are defined. One proposes empirical relationships to approximate the observed volume change with time and with the number of applied cycles, within the densification regime. Concerning the SPE’s, different kinds of sinking curves are observed depending on the regime of vibration ; they differ from the early results of Barkan (1962) which only presented “equivalent” Stokes sinking profiles.

INTRODUCTION

Definition

Vibrodriving is a technique for driving profiles as piles, tubes or sheet-piles swiftly into the soil by transmitting to the driving unit a longitudinal vibrating motion of predetermined frequency and displacement amplitude. The vibrations result in a reduction of the ground resistance, allowing penetration under the action of a relatively small surcharge force (Rodger and Littlejohn, 1980).

Historical development

Historically, it seems that the vibratory driving technique appeared simultaneously in the 30’s in Germany and in the former URSS. Pavyluk, who studied the effect of vibrations maintained on soil, introduced in 1931 the concept of driving profiles using vibrations. This work was later reported by Barkan (1962), who studied the effects of vibrations on the mechanical properties of soil, showing that the vertical vibration of a pile remarkably decreases the skin friction of surrounding soil.

In the 50’s, vibrodriving was used for the first time for the installation of about 4000 steel sheet-piles 9-12 meter long during the construction of the Gorki Hydroelectric Station (now called Nizhny Novgorod Hydroelectric Station, located in Russia). Using vibratory driver BT-5 (working between 38 and 45 Hz), the production rates reached were 11m in two or three min. Encouraged by the previous successes obtained by the Russian constructors, major manufacturers of vibrators were developed in Germany, USA, France, The Netherlands and Japan. Unfortunately, if great progress has been made in the technological development of various vibratory hammers (Holeyman, 2000) adapted with engineering ability to different types of soil and specific piles (Rodger and...
Littlejohn, 1980), due to a lack of fundamental understanding of the complex power pack-vibrator-profile-soil interactions, the vibro-driveability of a given profile into a given soil remains difficult to estimate.

**Engineering issues**

Figure 1 illustrates the vibrodriving of a pile on a typical site. Four major actors play a role in the mechanics of the vibratory driving process: the pile to be driven, the selected vibrator, the power generator, and the imposed soil conditions. Indeed, if it is obvious that soil properties influence the efficiency of the process, it also depends on the profile geometrical and mechanical properties (Rao, 1993 and Wang, 1994), on the vibratory hammer capacity in terms of the weight of the non-vibrating part of the vibrator and of its eccentric moment (Holeyman et al., 1996) and on the effective power supply provided by the hydraulic group (Holeyman and Whennam, 2008). In practice, the two parameters used to define the range of application of the method are the displacement amplitude and the angular frequency of the vibrator.

In the vibrodriving process, constructors are interested by mastering the following engineering issues, illustrated on Fig. 2. In spite of the numerous field studies conducted to determine the long-term bearing capacity of the vibratory driven pile, its evaluation remains a challenge at the present time (Russian Standard SNIP II-B.5-67, Polish Standard PN-83/B-02482, RGCU Vibratory pile-driving technical guide 2006). Research is needed to establish a vibratory testing procedure allowing the determination of the long-term bearing capacity from the monitoring of the vibratory driving.

There is the question of the vibro-driveability depending on the soil resistance to vibratory driving. If this latter can be calculated considering constant degradation parameters depending on the types of soil and based on soil investigations results (Holeyman, 1996 and 2000), there is a lack of experimental data to confirm this approach.

The coupled influence of the displacement amplitude and frequency on the driving velocity of the profile into the soil remains relatively misunderstood. Indeed, vibrodriving is used more and more to insert piles or sheet-piles into the soil, the required acceleration and frequency to enforce penetration still remain difficult to estimate. It is not always necessary to operate the hydraulic group at the maximum energy consumption to obtain the maximum penetration velocity.

Other engineering issues concern the soil vibro-compaction and the change in strength parameters in the near field around the vibratory driven pile. Finally, there is the question of mastering the propagation of vibrations to the surrounding soil to avoid vibratory nuisance (noise pollution or structural damages) to the environment.

**DEGRADATION PHENOMENA DURING VIBRATORY-DRIVING**

**Multiscale soil behaviour**

If the pile can be described by its material and geometrical properties including the consideration of its transversal and flexural properties and if the vibrator and the power pack can be chosen in function of a particular operational range, the interaction between these actors and the surrounding soil is more complex. The description of the behaviour of the soil under large cyclic strains is really complicated. Once the soil along the profile is vibrated with sufficient energy, the soil structure degrades and continuum modelling becomes less suited. It requires a multiscale computational framework.

In the far field, the elastic properties of soil govern its behaviour. The soil structure is not much modified. The energy transmitted by the vibrated pile is radially propagated and geometric damping is observed. Geometrical damping does not correspond to a dissipation phenomenon but the energy of the wave is simply redistributed on a growing surface when it moves away from the source. Equations of waves propagation in an elastic half-space can be considered (Achenbach, 1973).

In the near field, the soil is subjected to large distortions and its mechanical properties are progressively reduced until the zone located along the vibrated pile where soil is strongly degraded.

Fig. 1. Steel pile driven with vibrator in Belleville (F) (http://www.leductp.com/).
Figure 3 illustrates these phenomena. \( R \) is the radius of the vibrated pile and \( G \) is the tangent shear modulus of the soil. In the linear elastic zone or the far field, the shear modulus of the soil remains unchanged in spite of the transmission of the vibrations: \( G_{FF} = G_{init} \). In the non linear elastic zone or near field, the shear modulus is progressively reduced until a residual value. Along the profile, a slip zone is postulated where important convective movements of the strongly remolded soil take place. In this plastic zone, one thinks it becomes difficult to define a shear modulus (Gazetas and Dobry, 1984, El Naggar and Novak, 1994 and Michaelides et al., 1998).

The way the soil degrades depends on its nature. Indeed, different physical processes govern cyclic soil structure degradation in function of its geological origin. Cohesive soils subjected to cyclic shearing degrade because of the fatigue of the soil skeleton (Vucetic, 1994). Geometrical, hysteretical and viscous damping can occur depending on the magnitude of the shear strain or shear strain rate. The shear modulus is degraded in a way depending on the cyclic shear strain amplitude and on the plasticity index (Vucetic and Dobry, 1991). Viscosity effects depend on the strain rate and plastic index of the soil (Dobry and Vucetic, 1987). The readers can refer to Holeymman (2000) for more considerations about the cyclic behaviour of cohesive material and of saturated granular material exposed to vibratory driving. The present paper mainly focuses on cohesionless soil dynamic behaviour.

**Cohesionless soil degradation under cyclic loading**

Because of its granular nature, cohesionless soil behaviour is difficult to categorize. It is a geometrically complex assemblage of grains of various sizes and shapes resulting in a particular granulometry and a presence of voids defining its porous character. This pore space can be filled with some liquid according to its degree of saturation.

Many researches have been dedicated to strength degradation of cohesionless soils under large cyclic strains due to the increase of the pore pressure (Seed and Lee, 1966 and 1967, Casagrande, 1971 and Castro, 1975). Liquefaction flow and cyclic mobility phenomena have been highlighted to explain saturated sand behaviour.

Considering dry sand subjected to large cyclic strains, one argues it is possible to draw an analogy with cyclic mobility phenomena. For the purpose of investigating this question, one is initially interested in liquefaction phenomena.

Under drained conditions, if one subjects a saturated cohesionless soil to large cyclic shearing, there is an immediate volume reduction. Under undrained conditions, there is no change in volume and liquefaction or cyclic mobility phenomena can occur depending on the initial void ratio of the sand or of other granular materials, as illustrated on Fig. 4 (Castro and Poulos, 1977).

**Liquefaction flow phenomenon.** Liquefaction will be the result of undrained failure of a fully saturated highly contractive loose sand ending on the critical state line in continuous flow condition with large shear distortion. Upon reaching a critical state, the soil can flow at constant void ratio, constant effective mean stress and constant deviatoric stress.

**Cyclic mobility phenomenon.** Cyclic mobility only arises when a fully saturated dilative sand is subjected to cyclic loading. The effective mean stress can be ultimately reduced to zero because the pore pressure rises due to the undrained conditions. Then, the soil behaves as a fluid. Contrarily to the liquefaction phenomenon in loose sands, the strength degradation is stopped when cyclic loading is interrupted. Indeed, one can consider that the volume reduction (in drained conditions) or the increase of pore pressures (in undrained conditions) observed in cyclic loading are the expression of the same phenomenon, the irreversible tendency for a...
particular granular skeleton to reorganize itself into a denser arrangement when subjected to large amplitude cyclic loading.

Figure 5 presents the change in granular structure during undrained cyclic loading. At the microscopic scale of the grains, amplitude and orientation of contact forces between the grains are modified by the increase of the pore pressure resulting from the incompressibility of the interstitial fluid which cannot escape from the assemblage. Then, the chains of forces can be burst resulting in a particular liquefied state wherein particles become more free to move.

The macroscopic expression of this behaviour can be considered with the help of the intrinsic Coulomb failure criterion:

$$\tau_{ss} = (\sigma - u) \tan \phi$$

(1)

where $\tau_{ss}$ is the ultimate shear strength (kPa), $\sigma$ is the normal stress (kPa), $u$ is the pore pressure (kPa) and $\phi$ is the angle of friction ($^\circ$). The index $ss$ identifies saturated sand. $(\sigma - u)$ is the effective normal stress or intergranular stress. There is no cohesive term because of the cohesionless nature of the sand. If $u$ increases and becomes equal to or greater than $\sigma$, $\tau_{ss}$ vanishes.

Vibro-fluidization phenomenon. When vibrations are applied to a dry granular soil, one observes also a degradation of the granular skeleton, but there is no more interstitial fluid to explain the decrease of the shear strength of the material observed during cyclic loading. As in saturated conditions, it is possible to approach the physics of the phenomenon considering the microscopic point of view. When a dry granular material is vibrated, the contact forces between the grains can be progressively reduced, depending on the amplitude of vibrations. The chains of forces are interrupted and they can no longer transmit the shear stresses through the assemblage. The decrease of the lifetime of the contacts leads to a particular state wherein the sand behaves like a fluid. The soil is then vibro-fluidized in the absence of an interstitial fluid.

L’Hermite and Tournon proposed a theory to express this behaviour in a macroscopic point of view (as reported in Kolmayer, 1970). During cyclic loading, the vibrations would lead to the emergence of a “shaking pressure”, $\sigma_S$, which tends to decrease the normal pressure in a way similar to the pore pressure in presence of interstitial fluid. When the normal pressure is smaller than the shaking pressure, the shear strength of the sand gets close to zero. When the normal pressure becomes greater than the shaking pressure, the shear strength of the sand can increase. Hence, in dry conditions, the shaking pressure can be substituted for the pore pressure on Fig. 5. The intrinsic curve of the vibrated sand becomes:

$$\tau_{ds} = (\sigma - \sigma_S) \tan \phi$$

(2)

where $\tau_{ds}$ is the ultimate dry shear strength (kPa) and $\sigma_S$ is the shaking pressure (kPa). The index $ds$ identifies dry sand.

Hence, liquefaction or cyclic mobility in saturated sands and vibro-fluidization phenomenon in dry sands present points in common not only at the microscopic scale but also considering the expression of the intrinsic Coulomb failure criterion. Finally, all these phenomena contribute to the success of the vibrodriving process.

Many earthquakes have stimulated ample research on the liquefaction potential of saturated sands, in the field of seismic engineering. Others have succeeded in harnessing soil degradation as a result of cyclic straining and liquefaction, in the field of foundation engineering. By contrast, there is always a lack of consistent experimental data to illustrate the vibro-fluidization of dry granular soil during cyclic loading.

Hence, the present study concentrates on strength degradation of vibrated dry granular soils. It notably describes experiments on vertically shaken dry cohesionless sand. It displays the evolution of porosity during shaking, recirculation of particles.
and the results of sphere penetration experiments based on the conceptual work of Barkan (1962).

Effects of vibrations on internal friction of the sand

For the purpose of investigating the influence of vibrations on sand shear strength, several studies have been performed to characterize the influence of vibrations on the coefficient of internal friction in deviatoric conditions (as reported in Mogami and Kubo, 1953, Barkan, 1962, Youd, 1967, Ermolaev and Senin (1968a) and Kolmayer, 1970). If the friction coefficient was generally shown to decrease with acceleration amplitude, the results of these studies must be considered with caution due to the uncertainties concerning the experimental setup and because of the different procedures followed by the aforesaid authors. Moreover, results must be commented considering the directions of the applied vibrations. As an example, figure 6 presents the influence of vertical acceleration on the friction coefficient, $\tan \phi$, and on the shaking pressure, $\sigma_s$, for shear test results on dry sand, using vertically vibrated shear box (Kolmayer, 1970). $\Gamma = a/g$ (\text{m/s}^2) is the non dimensional acceleration where $a$ (\text{m/s}^2) is the acceleration amplitude and $g$ (\text{m/s}^2) is the gravity acceleration. The experimental setup of Kolmayer is illustrated on Fig. 7.

Considering the aforementioned studies, the governing parameter of the shear strength reduction in presence of vibrations seems to be the acceleration of the vibration, represented by $\Gamma$. If the influence of vibrations on soil resistance was highlighted in this section, the previous discussions do not encompass the progressive penetration of the profile into the soil during the vibrodriving process.

![Image of Fig. 6. Influence of the acceleration amplitude on the internal friction coefficient and on the shaking pressure of dry sand, after Kolmayer, 1970.]

Effects of vibrations on interface friction of the sand

The soil shear strength resisting the pulling out of a vibrating steel sheet-plate has been investigated by Russian researchers. Firstly, they considered the study of the interface friction of saturated sand on a vibrated sheet-pile which had been previously installed. Figure 8 presents the influence of the acceleration amplitude of the sheet-pile on the sandy soil friction resistance, measured on the lateral surfaces of straight web or U-shaped sheet-piles during vibro-extracting process (performed by Preobrajenskaïa and reported by Barkan 1963). At low amplitude of acceleration, one observes an effective decrease of the sand friction resistance. If the increase of the acceleration excitation allows a reduction of the effective friction applying to the lateral surfaces of the sheet-piles, beyond a threshold value, the lateral frictional resistance reaches an asymptotic value depending on the soil and sheet-piles properties. Then, there is no more influence of the acceleration amplitude on the vibro-extracting process. One assumes that the sand has reached its vibro-fluid limit.

BARKAN’S CONCEPTUAL MODEL (1962)

A major part of a pile or profile driving resistance usually comes from soil skin friction, which can be considered as the sum of “dry” friction and “viscous” friction depending on velocity. Barkan (1962) showed that internal friction resistance tends to vanish and that a granular soil starts behaving like a viscous fluid when subjected to intense vibrations. Following his own observations, this soil property could be expressed in terms of the coefficient of “vibro-viscosity”. Barkan (1962) attempted to determine this latter with the help of the penetrating sphere method considering the well-known Stokes law.

Barkan designed the sphere penetration experiment (SPE) to investigate this phenomenon and establish the coefficient of vibro-viscosity (Fig. 9a). He recorded the sinking velocity of a metallic sphere into a vertically vibrated container filled with sand to the height of 30 to 35 cm. The container, 30 by 30 cm in cross section, 40 cm height, was placed on a vibratory platform enforcing the vertical vibrations. Various “bias” loads could be applied to the sphere by means of a loading system. The penetration of the sphere was recorded with the
help of a recorder and counterweights. The sand used was white quartz sand with a grain size ranging between 0.2 and 0.5 mm. The moisture content of this latter was close to zero.

Before performing a sphere penetration experiment, the sand was vibrated until its minimum void ratio, approximately 0.5 (-), was reached, leading to a volumetric mass of soil about 1.77 (gr/cm³).

The use of the Stokes method assumes the presence of a penetration phase wherein the sinking velocity of the sphere is constant. If the steady driving force acting on the sphere only comes from gravity, Stokes law can be applied to the sphere penetrating at a constant rate into vibrated sand, considered as a viscous medium:

\[
\mu V = \left( \frac{2}{9} \right) r^2 (\gamma_1 - \gamma_2) \tag{3}
\]

where \(\mu\) (Pa.sec) is the “liquid” medium viscosity, \(V\) (m.sec\(^{-1}\)) is the steady sinking velocity, \(\gamma_1\) (N.m\(^{-3}\)) is the equivalent sphere unit weight and \(\gamma_2\) (N.m\(^{-3}\)) is the “liquid” unit weight.

From the sphere penetration experiments results (Fig. 9b), the sinking velocity of the sphere seemed to obey Stokes law allowing the determination of the coefficient of vibro-viscosity, \(\mu\). The inverse of this latter was shown to vary linearly with the relative level of acceleration:

\[
\frac{1}{\mu} = b (\Gamma - \Gamma_0) \tag{4}
\]

where \(b\) (Pa\(^{-1}\).sec\(^{-1}\)) is an empirical parameter and \(\Gamma_0\) (-) is a threshold value approximately equal to 1.5. Upon exceeding this threshold value, there is a significant increase of \(1/\mu\) and Barkan considered that soil enters in a vibro-fluidized state and behaves like a viscous liquid. Nevertheless, a number of critical observations can be formulated regarding the early results of Barkan (1962).

According to our own experiments discussed in the following sections, the shape of the penetrating logs (Stokes law for all Barkan’s results) could depend on the one hand, on the vibration rate applied to the sand (coupled influence of the displacement amplitude and frequency) and on the other hand, on the initial void ratio of the granular medium in the container before beginning the sphere penetration experiment. Below the threshold value, \(\Gamma_0\), there is no information about potential sphere refusal conditions, as Stokes’ law does not entail such limitation.

Considering the previous study of Ermolaev and Senin (1968a) on shear strength degradation under vertical vibrations, one can be surprised by the absence of progressive states depending on acceleration amplitude, \(\Gamma\).

There is a lack of indication about the vibrated sand behaviour. Barkan (1962) did not describe the movement of the soil particles. Is the sand submitted to general convective motion or is there just a downward segregation movement of the sphere in the vibrated sand which is in settlement?

For the purposes of investigating these questions, a new SPE apparatus was designed at UCL.

NEW SPE APPARATUS

Figure 10 presents the SPE apparatus designed and constructed at UCL. In the lower part, a cylindrical transparent polycarbonate container (52 cm height and 40 cm diameter) is vertically actuated by a servo-controlled MTS hydraulic jack. The movement programmed to the container is sinusoidal. Samples of 65 kg of Fontainebleau sand are prepared dry by pouring, resulting in a volumetric mass close to 1.47 gr/cm³ or initial void ratio, \(e_{\text{init}}=0.8\) (-).
In the upper part, the sphere penetrometer is free to move only vertically. The penetration of the sphere was recorded with the help of a 500 mm range inductive HBM standard displacement transducer. The sphere diameter is 28.58 mm. The total mass of the sphere penetrometer is 1.57 kg.

The Fontainebleau sand is a fine quartz sand with uniform grain size (uniformity coefficient of Hazen, $C_u=1.6$ and $d_{50}=0.19$ mm). The grain-size distribution ranges from 0.1 to 0.4 mm. The grain shape can be characterized as subangular. The volumetric mass of the sand is 2.642 g/cm$^3$. The minimum volumetric mass of dry sand, reached with loose bulk density test, is $\rho_{\text{min}}=1.406$ g/cm$^3$, $\rho_{\text{max}}=1.702$ g/cm$^3$, $e_{\text{min}}=0.55$ (-). The critical state angle of friction, $\Phi_{\text{crit}}$, is expected to be close to 30$^\circ$.

Before performing sphere penetration experiments (SPE), one initially explores dry sand behaviour without inserting profile in the vertically vibrated container.

VERTICALLY VIBRATED SAND BEHAVIOUR

The considerations about vertically vibrated sand behaviour can be related to the engineering issues concerning the vibrocompaction of granular soils. This industrial process was introduced in Germany in the 30’s. Applied to granular media, it is more efficient than static compaction. The purpose of this dynamic compaction is generally improving deformation properties and shear strength of the soil. The result of the compaction is commonly related to soil density. Although research has been conducted on compaction, the physical mechanisms at work during vibrocompaction of sand are poorly understood and the description of the grains behaviour remains incomplete.

When cohesionless soil is vertically vibrated under gravitational field, several phenomena can occur. The following paragraphs concern the void ratio change with time and the void ratio change with the number of cycles applied to the container, for acceleration amplitude $\Gamma<1$. Then, one describes the sand particles behaviour during vibrations. Finally, one concentrates on the influence of the acceleration amplitude on the resulting dry density after vibrations.

Volume change with time

Ermolaev and Senin (1968b) have suggested a relationship between void ratio and time for vertical vibrations:

$$e(t) = e_\Gamma + (e_{\text{init}} - e_\Gamma) \exp(-\beta_t t)$$  \hspace{1cm} (5)

where $e(-)$ is the void ratio, $e_{\text{init}}(-)$ is the initial void ratio resulting from dry pouring, $t$ is the time (min), $e_\Gamma (-)$ is the minimum void ratio which can be reached with the imposed acceleration amplitude, $\Gamma (-)$, and $\beta_t (\text{min}^{-1})$ is a coefficient depending on the nature of the soil and acceleration amplitude. Barkan (1962) has suggested the following equation for $e_\Gamma$:

$$e_\Gamma = e_{\text{min}} + (e_{\text{init}} - e_{\text{min}}) \exp(-\alpha_t \Gamma)$$  \hspace{1cm} (6)

where $\alpha_t (-)$ is the index of vibratory compaction depending on the nature of the soil.

Figure 11 presents void ratio-time curves during vertical shaking for some experiments realized with the UCL-SPE apparatus. The displacement amplitude was set to $A=1.5$ (mm) for all the experiments. Initially, the void ratio quickly decreases. Then the volume change is slowing down and the void ratio leads towards an asymptotic value, $e_\Gamma (-)$, depending on the acceleration amplitude of the vibrations. The grey continuous lines are the estimated void ratios, computed using equations (5) and (6) with $\beta_t (-)$ and $\alpha_t (-)$ depending on the acceleration amplitude, $\Gamma$.

The expression (5) of the void ratio change with time and the expression (6) of the asymptotic void ratio, $e_\Gamma$, seem to be convenient for the description of our experiments. Nevertheless, the UCL observations allow noting that $\beta_t$ and $\alpha_t$ are directly dependent on the acceleration amplitude, $\Gamma$. Indeed, it is possible to describe the void ratio time curves with the following empirical expressions depending on $\Gamma$ and on constant empirical coefficients:
\[ e(t) = e_f + (e_{init} - e_f) \exp (-0.0888 \exp (2.6728 \Gamma)) t \]  \hspace{1cm} (7)

\[ e_f = e_{min} + (e_{init} - e_{min}) \exp(-0.93668\Gamma^2 - 0.2188 \Gamma) \]  \hspace{1cm} (8)

The grey dashed lines represent these relationships on Fig. 11.

**Volume change with number of cycles**

For the purpose of investigating the influence of the frequency, it is possible to describe the void ratio change with the number of cycles applied to the container. Figure 12 represents void ratio-number of cycles curves during vertical shaking for the same experiments. One proposes the following relationship to describe the volume change:

\[ e(-) = e_f + (e_{init} - e_f) \exp(-0.0003 \exp(1.882\Gamma)N) \]  \hspace{1cm} (9)

where \( N \) is the number of cycles applied to the container and \( e_f \) is given by equation (8). These relationships are represented by grey lines on Fig. 12.

**Sand behaviour during vibration**

**Densification range.** When non-cohesive soil is vertically vibrated under gravitational field, Mogami and Kubo (1953) and Barkan (1962) assert that the volume change depends mainly on the acceleration of the vibrations. The influence of acceleration amplitude was explored with the UCL-SPE apparatus and vertically vibrated sand behaviour was observed during shaking. These experiments lead to the following preliminary observations. If the peak acceleration is less than 1g, there is a densification of the sample. The efficiency of this densification depends on the initial void ratio of the sample and on the acceleration amplitude. The equations (5) to (9) appear to be solely valid within this densification range.

**Instability surface range.** When acceleration amplitude reaches 1g, several phenomena come into competition. The first phenomenon is the free-fall-impact condition. When the acceleration is greater than 1g, the sand cannot fully follow the downward motion of the container and goes into free-fall. When the sand reaches the bottom of the container, there is an impact and a propagation of stresses within the granular medium in the form of force chains. This pattern should result in efficient densification.

Nevertheless, as observed in the UCL experiments, when the acceleration is increased beyond 1g, there is an instability in the sand heap leading to the emergence of an inclined free surface. It is characterized firstly by a continuous flow of particles rolling down the inclined surface and secondly by a convective transport inside the vibrated sample which recirculates sand grains at the top of the heap, as observed previously by Evesque and Rajchenbach (1989). The cause of this particular phenomenon seems to be the convective motion accompanied by an upward wave pulse.

During convection, the granular medium convects in response to vertical vibrations (Knight et al., 1993). There is an upward granular flow in the center of the container and a downward granular flow in a thin veneer along the sides. This phenomenon is due to the inelastic frictional interactions between the grains and the walls of the container (Grossman, 1997).

When the container is raised up, the gravitational acceleration is opposed to the movement of the granular material and there are few relative movements between the sand particles. When the container starts going down, there is a relative acceleration between the grains and the container (\( -1 \)g). Then, the grains are momentarily separated from the container. There are more relative movements between particles and there is a progressive inclination of the free surface which could be due to an upward wave, as explained in Pak and Behringer (1993).

Indeed, when the granular matter is vertically vibrated, there is a dissipation of the introduced kinetic energy by multiple inelastic interactions between the grains. This loss of energy depends on the particles assemblage. The dissipation of energy...
per unit length in the vertical direction varies in function of the angular position and depends on the number of grains in contact at different locations. This inevitable asymmetry, in the structure of the sample and in the way the kinetic energy is dissipated, would be at the origin of the emergence of an upward wave impulse in the container.

The propagation of this wave can be explained in this way. There is a supply of sand at the top of the bump by the avalanches. In contrast, at the low edge of the bump, the slope becomes steeper and the avalanche rate is higher in comparison with the uphill edge of the bump leading to the spreading of the pulse. This complex process leads to the forming of an inclined plane free surface in the vertically vibrated container.

The angle of the free surface is limited by the nature of the sand particles. During the upward phase, the sand is compacted and behaves like a solid. The slope of this latter can not exceed a maximum dynamic angle which mainly depends on the nature of the grains. When this maximum angle is reached, the excess of matter flows in continuous avalanches. In this paper, the term avalanche references superficial landslide.

Figure 13 illustrates the emergence of the slope in a container vertically vibrated with $\Gamma$ close to 1, in the course of the vibration. Initially, the convection phenomenon is initiated along the walls (Fig. 13a). Then, due to the asymmetry of the assemblage, upward waves are generated and they lead to the emergence of an inclined free surface in the container. The convective rolls circulate sand grains at the top of the emergent heap. Intermittent avalanches appear at the free surface (Fig. 13b). Then, due to the upward waves, there are successive bumps under the inclined free surface covered with continuous avalanches. The slope progressively develops (Fig. 13c). Finally, the instability surface regime is developed. Keeping the balance between the upward waves and the maximum dynamic angle, avalanches determine the slope of the inclined free surface (Fig. 13d).

**Vibro-fluid range.** If the acceleration amplitude is still increased beyond a value about 1.75g, the inclined free surface is progressively dislocated. One observes an impressive bulge which tends to expand the whole sample. The sand becomes fully vibro-fluidized. The convection still remains but it presents a more chaotic character. Additionally, a new state of the horizontal free surface can develop with the emergence of particular geometrical shapes intermittently accompanied by chaotic grain saltation, as observed by Melo et al. (1994).

**Dry density after vibration**

Figure 14 illustrates the relationship between sand density after vibrations and vertical acceleration for the UCL experiments. In the densification range, one observes a progressive increase in the final density until the acceleration amplitude reaches 1g. Beyond this value, the convective process accompanied by the inclined free surface tends to dilate the vibrated granular matter and the dry density after vibrations decreases progressively to reach a value oscillating between 1.55 and 1.60 when the sand is vibrated in the vibro-fluid range.

In Fig. 14, one compares our results with the D’Appolonia D. J. and D’Appolonia E. observations (1967). They have conducted laboratory tests where an open container of air-dry dune sand is placed upon a shaking table and subjected to vertical vibration. If the peak acceleration of the vibrated container is less than 1g, one can surmise a poor densification of the sample. But this observation is contradicted by the Barkan (1962), Ermolaev and Senin (1968b) and UCL observations (2009). Beyond 1g, it seems that there is an important densification of the D’Appolonia D. J. and
D’Appolonia E. sample. According to these authors, it can be explained by the free-fall-impact condition. Beyond that range of peak accelerations, there is a densification of the sample. Unfortunately, they do not describe the motion of the sand particles within these acceleration ranges.

According to the UCL observations, vertical accelerations, in excess of 1g, are counter-productive to the effective dynamic compaction of the sand. Indeed, under vertical vibrations, there is an optimum acceleration close to 1g allowing optimum dynamic compaction (Prakash and Gupta, 1967 and Kolmayer, 1970).

Contrarily, under horizontal vibrations, there is no instability phenomenon once the acceleration amplitude reaches 1g. The increase of the acceleration, beyond this value, leads to an improved compaction of the dry sand (Barkan, 1962, Prakash and Gupta, 1967, Greenfield and Misiaszek, 1967 and Youd, 1967).

SPHERE PENETRATION EXPERIMENTS (SPE)

Field of application of the SPE’s

It is necessary to define the field of application of sphere penetration experiments. As a first approach, sphere penetration experiments will be performed in vibrated sand sample presenting a horizontal surface, hence, within the densification and vibro-fluid ranges. Indeed, the SPE, performed in the surface instability range, are strongly disrupted by the convective rolls present in this range.

SPE’s procedure

In Barkan’s experiments, the sand seems to be prepared for all sphere penetration experiments following the same procedure. It is always vibrated with acceleration allowing minimum void ratio, $e_{\text{min}} = 0.5$ (+). The UCL procedure is different in that regard, depending on the vibration range.

For the UCL-SPE’s conducted within the densification range, the sand is initially vibrated until the equilibrium (steady-state) void ratio, $e_f$, corresponding to the chosen acceleration amplitude, $\Gamma$, is reached. Then, vibrations are stopped and the sphere is lowered onto the sample resulting in a quasi-static penetration. Once static equilibrium is reached, vibration of the container is resumed and dynamic penetration is recorded.

For the UCL-SPE’s performed within the vibro-fluid range, the sample is vibrated until the vibro-fluidization is complete. Then the sphere is lowered onto the vibrated surface of the sample and its dynamic penetration recorded.

Results of the new SPE’s

Figure 15 presents sinking logs for several amplitude of vibration. The displacement amplitude is 1.5 mm for all the SPE’s. The frequency varies between 9 and 25 Hz. The resulting acceleration amplitudes are noted on Fig. 15. Sphere position is defined as the vertical distance between the lower part of the sphere and the bottom of the container.

At low acceleration amplitude ($\Gamma < 1$), the sinking logs seem to follow an exponentially decreasing profile (Denies et al., 2009). The rate of penetration depends on the vibration parameters and on the initial void ratio of the sample. The motion of the sphere is sinusoidal and presents a frequency and a displacement amplitude close to the imposed motion of the container with a residual sinking component.

At high acceleration amplitude ($\Gamma > 1.75$), after a short transient sinking phase resulting from its initial position (lowered onto the highly tumultuous horizontal free surface), the sphere reaches an equilibrium penetration. The fluctuations, around this equilibrium position, are due to the variation of the collisions and dry frictions with the upward convective flow of surrounding particles. Hence, at the equilibrium height, the weight of the sphere penetrometer seems to be balanced by a “buoyant” force augmented by the flow of surrounding sand grains. It is to note that the height of equilibrium does not depend on the initial void ratio of the sample.

Moreover, in the vibro-fluid regime, if the sand is fully vibro-fluidized and seems to present the consistency of a liquid, the sinking logs do not follow Stokes law, as initially reported by Barkan (1962). The sinking logs are very similar and the height of equilibrium seems to become independent of the acceleration amplitude in contradiction with the early results of Barkan (1962). The concept of the coefficient of vibro-viscosity of the vibrated granular soil needs to be reappraised.

In the vibro-fluid range, the governing parameters of the equilibrium height of the sphere are then its apparent mass and its diameter.

CONCLUSIONS AND PERSPECTIVES

This paper concerns vibrocompaction and vibrodamping processes applied to granular soils. It concentrates on the shear strength degradation of the sand subjected to vertical vibrations.

Vibrofluidization of dry granular soils is discussed in connection with cyclic mobility phenomenon in saturated sands.

Considering the influence of vibrations on the sand shear strength in deviatoric conditions, the friction coefficient ($\tan \phi$) is generally shown to decrease with acceleration amplitude. But the results of the various studies must be considered with caution due to the uncertainties concerning the number of
When cohesionless soil is vertically vibrated under gravitational field, the UCL experiments performed on dry Fontainebleau sand distinguish three different sand dynamic behaviours depending on the acceleration amplitudes: the densification behaviour ($\Gamma<1$), the surface instability behaviour ($1<\Gamma<1.75$), and the vibro-fluid behaviour ($\Gamma>1.75$). In the densification range, the volume change with time or with the number of applied cycles can be approximated by exponential relationships only function of the acceleration amplitude. Considering the dry density after vibrations, there is an optimum acceleration close to $1g$ allowing optimum vertical dynamic compaction.

Concerning the UCL-SPE’s realized under gravitational field, two kinds of sinking curves are displayed. An exponentially decreasing profile is observed in the densification range whereas in the vibro-fluid range, the sphere quickly sinks in the vibrated medium and finds an equilibrium penetration. The weight of the sphere penetrrometer is balanced by a “buoyant” force augmented by the flow of surrounding sand grains. Additional experiments, with different displacement amplitudes, will further characterize the influence of vibration parameters on sinking log in the both ranges.

The sinking logs of the UCL-SPE’s do not follow Stokes’ law curve, contrary to Barkan’s observations (1962). Hence, the concept of the coefficient of vibro-viscosity of the vibrated granular soil, proposed by Barkan and assuming a linear relationship between this coefficient and the acceleration, is to be reappraised.

If one considers the vibro-extracting experiments performed by Preobrajenskaïa, figure 8 demonstrates that increasing the acceleration of a sheet-pile actually allows a reduction of its effective lateral friction resistance. But beyond an acceleration threshold value, the lateral frictional resistance reaches an asymptotic value. As in the case of our sphere penetration experiments, once the sand has reached its vibro-fluid limit, there is finally no more influence of the acceleration amplitude on the vibratory driving resistance.

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