Influence of an impedance change on SRD computation by the Case Method

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ABSTRACT: Used during pile driving, the Case Method offers an immediate estimate of the soil resistance to driving (SRD) after each hammer blow. Although it is over 40 years old, it is still widely used in its original form in the offshore piling industry. Indeed, the estimated SRD, corrected for set-up effects, provides an indication of the pile static capacity. The Case Method requires measurements of force and velocity near the pile head as the hammer strikes the pile and produces an analytical form of the SRD, using a number of assumptions. One of them requires the pile to be of constant impedance (or cross section) across its length. Offshore driven steel pipe piles are usually very long and therefore composed of several pieces of different cross sections. Furthermore, the welding of these pieces also produces local variations of impedance of the pile. In this context, an improved version of the original Case Method is suggested to take into account a possible variation of impedance along the pile.

1 INTRODUCTION

The numerical application of the wave equation theory to a pile subjected to an impact (during driving or dynamic load testing) was introduced by Smith (1960) and is still vastly used nowadays (Holeyman 1992, Rausche et al. 2008).

The main assumption behind the classic wave equation analysis is that the pile is considered as a one-dimensional unconstrained rod. The impact of the hammer on the pile head creates a downward stress wave that propagates through the pile. For a pile behaving elastically, the waves generated by the application of a sudden axial force are axial body waves which follow the 1D wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial z^2} \quad (1)$$

where u = displacement function (downward positive), t = time, z = position (downward positive) and c = wave propagation speed, which is given by:

$$c = \sqrt{\frac{E}{\rho}} \quad (2)$$

where E = Young modulus of the pile and ρ = its density.

The solution of the wave equation (Eq. (1)) is the sum of downward and upward waves travelling at constant celerity c. These waves are described by Eq. (1) in terms of displacement but induce corresponding forces and velocities within the rod (Timoshenko and Goodier, 1970). The velocity (v) and force (F) waves at any given pile section upon arrival of the displacement wave can also be split into their downward (d) and upward (u) travelling components, i.e.:

$$v = v_d + v_u \qquad F = F_d + F_u \quad (3)$$

The normal forces are positive when compressive whereas velocities at any given cross section upon passage of the wave are positive downwards. Equilibrium conditions account for the normal force orientation relative to the boundary they are applied to.

The wave equation (Eq. (1)) is obtained by considering a pile without any external force acting on it. Nevertheless, in order to correctly describe the pile driving problem, shaft friction and base resistance have to be taken into account. Induced by pile movement, these soil reactions attenuate the descending initial wave and produce upward travelling waves. Soil resistance is usually modelled by considering a finite number of equally spaced forces along the pile. This simplification is also used in order to establish the Case Method, as discussed in Section 3.

Two relationships that will be used in the following sections are now introduced. From Eq. (1), we can obtain relationships between the force and the velocity, respectively, of a wave travelling down the pile and of a wave travelling up the pile:

$$F_d = Iv_d \tag{4a}$$
$$F_u = -Iv_u \tag{4b}$$

with the pile impedance being defined by the following equation:

$$I = \sqrt{E\rho}A = \frac{EA}{c} = \rho c A \tag{5}$$

where A = pile cross-sectional area.

Equations (4a) and (4b) express that when a single wave travels down the pile, force F_d and velocity v_d are proportional and have the same sign. On the other hand, when a single wave travels up the pile, force F_u and velocity v_u are proportional and have opposite signs.

A second set of relationships allows us to evaluate the upward and downward force wave from the measurable force and velocity at a given section along the pile:

$$\begin{aligned} 2F_d &= F + Iv \\ 2F_u &= F - Iv \end{aligned} \tag{6}$$

2 SOIL RESISTANCE TO DRIVING

Figure 1 schematizes the evolution of the static resistance of a pile before, during and after driving a given segment m. The pile is being redriven and already has some static capacity from being embedded in the soil. During the driving, i.e. between the beginning of redrive (BOR) and the end of driving (EOD), the static resistance decreases due to degradation of the soil surrounding the pile (e.g. in saturated soils, the degradation mainly results from the pore pressure build up). After a given number of blows, an asymptotic trend emerges in the form of an endurance limit, namely the "soil resistance during continuous driving" (SRCD).

Once the driving stops, a restoration process begins which is known as "soil setup". The pile capacity, if not redriven, will eventually reach the long term static soil resistance (SSR; Rausche and Hussein, 1999).

The static resistance developed by the pile during driving is called the soil resistance to driving (SRD).

Furthermore, the velocity imposed by the driving adds a "dynamic" term to the SRD.

The terms described above are not always univocally named in the literature. In this paper, the following definitions will be used: the displacement (respectfully velocity) induced resistance will be called the static (respectfully damping) resistance while the sum of the static and damping resistance will be named the total resistance.

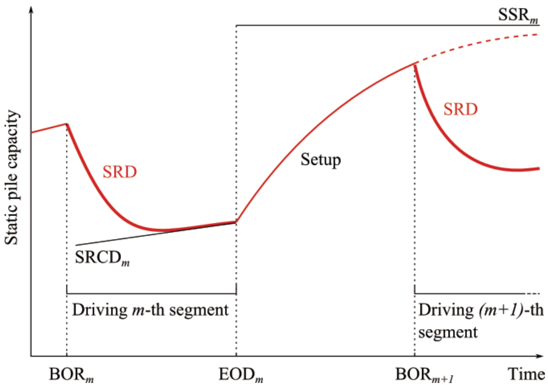


Figure 1. Evolution of the resistance offered by the soil with respect to time during two driving sequences.

The wave equation analysis enables us to assess the pile resistance by processing force and velocity signal measured at the pile head during driving.

Two particularities arise from using the data measured during pile driving. Firstly, the separation of the static and damping parts of the total resistance has to be carefully made to extract the static soil resistance (SRD). Secondly, the moment chosen to estimate the SRD is of paramount importance. The SRD computation using data from a blow following a long setup period will produce a closer value to the actual pile capacity, as it can be seen on Fig. 1.

3 THE ORIGINAL CASE METHOD AND ITS ASSUMPTIONS

The Case Method is an analytical formula which provides an estimate of the SRD based on force and velocity measurements at the pile top during an impact.

We consider a pile of length L. The force and velocity induced by the wave at depth z and time t will be noted, respectively, $F(z,t)$ and $v(z,t)$.

In order to set up the Case Method, Rausche et al. (1985) firstly assumed that forces in the form of soil resistances were concentrated at a finite number of locations along the pile. These resistive forces consist of a displacement dependent (static) part and a velocity dependent (damping) part. The static resistances follow an ideal rigid plastic behaviour.

The damping resistances follow the Smith (1960) shaft reaction model with the assumption that they solely depend on the toe velocity $v(L,t)$ at the time the wave initiated at t_0 arrives at the pile toe (at $t=t_0+L/c$), so the damping resistance can be expressed as follow:

$$R_d = j_c I v(L, t_0 + L/c) \tag{7}$$

where j_c = Case damping factor. The static resistance R_s expressed by Rausche et al. (1985) is the SRD.

Furthermore, the assumptions made by the Case Method promotors can actually be boiled down to the following stricter ones:

- The pile is considered as an elastic rod and the hammer blow axial,
- A soil resistance at depth z^* is activated at least at both times $t_0 + z^*/c$ and $t_0 + (2L - z^*)/c$ (for a rigid plastic behaviour, this assumption is usually encapsulated into a more demanding assumption, namely: the pile has to keep moving down during the time interval $2L/c$),
- The pile cross sectional area is uniform along the pile length.

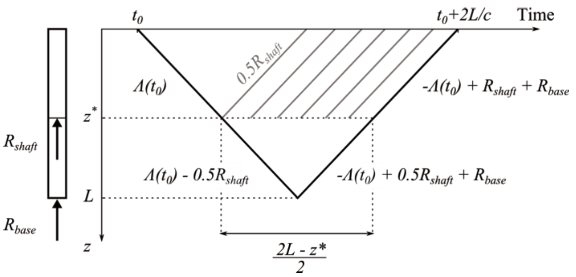


Figure 2. Pile subjected to a force boundary condition $F_d(0,t_0) = \Lambda(t_0)$. For purposes of clarity, there is only one application level (z^*) of the shaft resistance R_{shaft}.

As suggested by Salgado (2008), the setting up of the Case formula can be alternatively demonstrated in two steps. Let us suppose that the force wave induced by the hammer blow at pile head at time $t = t_0$ is $F_d(0,t_0) = \Lambda(t_0)$. Firstly, the expression of the upward force wave at the pile head at time $t_0 + 2L/c$ gives an expression of the total resistance (Fig. 2):

$$R = F_u(0, t_0 + 2L/c) + \Lambda(t_0) = R_s + R_d \tag{8}$$

It can be noted that resistance is positive in the upward direction. The second step consists in expressing the equilibrium of forces of an infinitesimal slice of the pile at its toe at the moment the wave generated by the hammer blow arrives (at $t = t_0 + L/c$, Fig. 3). Upon reaching the pile toe, the upward force wave must fulfil the vertical equilibrium condition:

$$F_u(L, t_0 + L/c) + \Lambda(t_0) - \frac{1}{2} R_{shaft} - R_{base} = 0 \tag{9}$$

Using Eqs. (3), (4) and (9), the particle velocity pile at pile toe is given by:

$$\begin{aligned} v(L, t_0 + L/c) &= \frac{1}{I} F_d(L, t_0 + L/c) \\ &\quad - \frac{1}{I} F_u(L, t_0 + L/c) \\ &= \frac{2}{I}\Lambda(t_0) - \frac{1}{I} R \end{aligned} \tag{10}$$

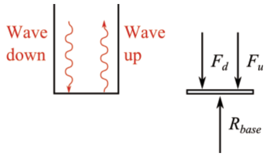


Figure 3. Forces acting the pile toe at time $t_0 + L/c$: the base resistance R_{base} and the forces F_d and F_u respectively created by the downward and reflected upward moving waves.

Using Eqs. (8), (10) and (7) yields a formulation of the SRD in function of measurements acquired at the pile head at times t_0 and $t_0 + 2L/c$, which is known as the Case formula:

$$\begin{aligned} R_s &= (1 + j_c) F_u(0, t_0 + 2L/c) \\ &\quad + (1 - j_c) F_d(0, t_0) \end{aligned} \tag{11}$$

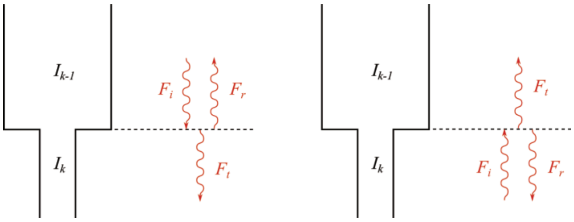
Eq. (11) is usually expressed in terms of force and velocity at the pile head by using Eqs. (6).

4 A MODIFIED CASE METHOD FOR PILES OF VARYING IMPEDANCE

One of the assumptions of the original Case Method consists in having a constant impedance along the pile (uniform cross section and one material composing the pile). The following sections propose a modified Case formula that takes into account the possible changes of impedance along the pile.

4.1 Wave modifications upon encountering a change of impedance

When a wave travelling along the pile encounters an abrupt change of impedance (Fig. 4), part of the incoming wave (*i*) is reflected (*r*) and part of the wave is transmitted (*t*). In order to quantify these two parts, the impedance ratio $i_k = I_{k-1}/I_k$ is introduced, where I_{k-1} and I_k are the impedances of the pile respectively above and below the *k*-th impedance change. The impedance itself is defined in Eq. (5).



(a) (b)

Figure 4. Incident (*i*), reflected (*r*) and transmitted (*t*) waves when coming across an impedance discontinuity.

At the impedance discontinuity, the compatibility of displacements and the axial equilibrium must be satisfied, yielding the transmission and reflection coefficients relative to the incoming wave. For a downward force wave (Fig. 4a), the amplitude of the reflected and transmitted force waves are given by:

$$F_r = \frac{1-i_k}{1+i_k}F_i \qquad F_t = \frac{2}{1+i_k}F_i \tag{12}$$

And for an upward incoming wave (Fig. 4b), these are:

$$F_r = \frac{i_k-1}{1+i_k}F_i \qquad F_t = \frac{2i_k}{1+i_k}F_i \tag{13}$$

For the modified Case formula and for *n* impedance changes, the definition of the Case damping factor j_c (Eq. 7 for the original formula) needs to be slightly tuned:

$$R_d = j_c I_n v(L, t_0 + L/c) \tag{14}$$

where I_n is the impedance after the *n*-th impedance change (i.e. the impedance of the pile toe).

4.2 Modified Case Method

The development of the suggested modified Case Method is presented herein for a pile with one impedance change ($n=1$) at depth $z = z_s$ (Fig. 5). To keep notations concise, the impedance ratio i_1 will be noted *i*.

In order to produce an analytically accessible form of the SRD, the following assumption is made: all of the pile resistance is lumped at its toe, thus $R = R_b$.

The modified Case formula is obtained by applying the same two steps used in Section 3. The expression of the upward force wave at the pile head at $t_0 + 2L/c$ yields:

$$R = \frac{1+i}{2i}F_u(0, t_0 + 2L/c) + \frac{2}{1+i}\Lambda(t_0) - \frac{1-i}{2i}\Lambda(t_s) \tag{15}$$

where t_s = time at which originated the wave arriving at the pile top after reflecting against the impedance change at point $z = z_s$ (Fig. 5).

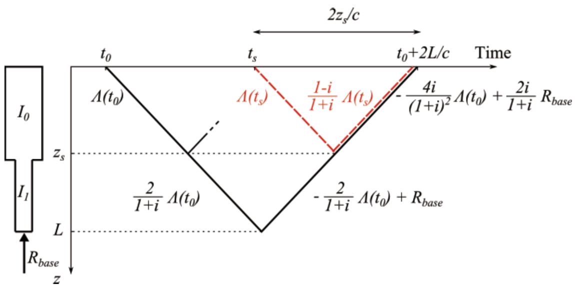


Figure 5. Force waves arriving at time $t_0 + 2L/c$ at pile head for a pile with one impedance change at $z = z_s$.

Secondly, the velocity at pile base after the wave initiated at t_0 has time to arrive can be expressed as follows:

$$v(L, t_0 + L/c) = \frac{1}{I_1}\frac{4}{1+i}\Lambda(t_0) - \frac{1}{I_1}R \tag{16}$$

As for the original Case formula, the estimated SRD is obtained by using Eqs. (15), (16) and (14):

$$R_s = (1+j_c)\frac{1+i}{2i}F_u(0, t_0 + 2L/c) + (1-j_c)\frac{2}{1+i}F_d(0, t_0) - (1+j_c)\frac{1-i}{2i}F_d(0, t_s) \tag{17}$$

Eq. (17) is the modified Case formula for one impedance change (characterized by the impedance ratio i) allowing to estimate the SRD from force and velocity measurements at the pile head (using Eqs. 6) at times t_0, t_s and $t_0 + 2L/c$.

The case of n impedance changes has been addressed in De Chaunac and Domange (2011) and is undergoing refinement towards another publication.

5 INFLUENCE OF LUMPING THE RESISTANCE TO THE TOE

Let us consider a pile with one impedance change and with one resistive force R_{shaft} which is located above the impedance change. If the toe lumping assumption is used, the effect of R_{shaft} at pile head at time $t_0 + 2L/c$ is:

$$\frac{2i}{1+i} R_{shaft} \tag{18}$$

If the toe lumping assumption is not used, the influence of the shaft resistance at pile head is:

$$\frac{1}{2} R_{shaft} + \frac{2i}{(1+i)^2} R_{shaft} - \frac{1}{2}\frac{1-i}{1+i} R_{shaft} \tag{19}$$

By comparing these two expressions, the error made by taking the toe lumping assumption varies from -15% to +20% for impedance ratios i ranging from 0.5 to 2.

6 PARAMETRIC STUDY: INFLUENCE OF THE DEPTH OF THE IMPEDANCE CHANGE

The following section presents a numerical parametric study that compares results obtained with the original Case Method and the modified one. The numerical tool used was created by the authors using Matlab. The program solves the wave equation in an unconstrained rod by the method of characteristics and allows imposing impedance changes and resistive forces along the pile. The models used for the resistive forces are consistent with the Case Method hypothesises, i.e. the resistive forces are activated once the wave arrives at their level ($t_0 + z^*/c$) and until the wave reflected at pile toe passes through again (i.e. at time $t_0 + (2L - z^*)/c$). The details of the numerical program can be found in De Chaunac and Domange (2011).

In order to provide a certain amount of realism, (a) the pile used in the modelling has characteristics of a typical offshore steel pipe pile (specifications of the pile and of the driving equipment are given in Table 1) and (b) the force boundary condition imposed at the pile head is an analytical function of the force imposed by a hammer blow on a semi-infinite pile. A graphical output of the numerical program is provided in Fig. 6 for a free pile with only one impedance change at depth $z_s = 50\,\text{m}$.

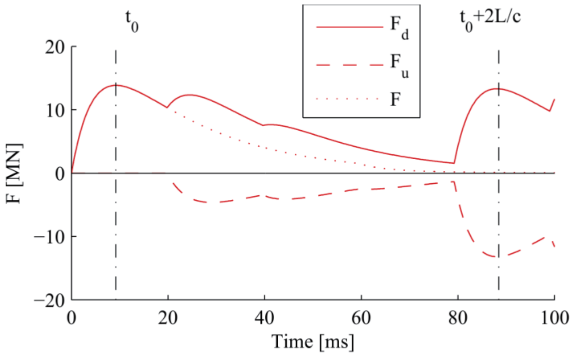


Figure 6. Upward and downward force wave computed at pile head. The pile characteristics are given in Table 1. No resistive force and one impedance change (i=2) at $z_s = 50\,\text{m}$.

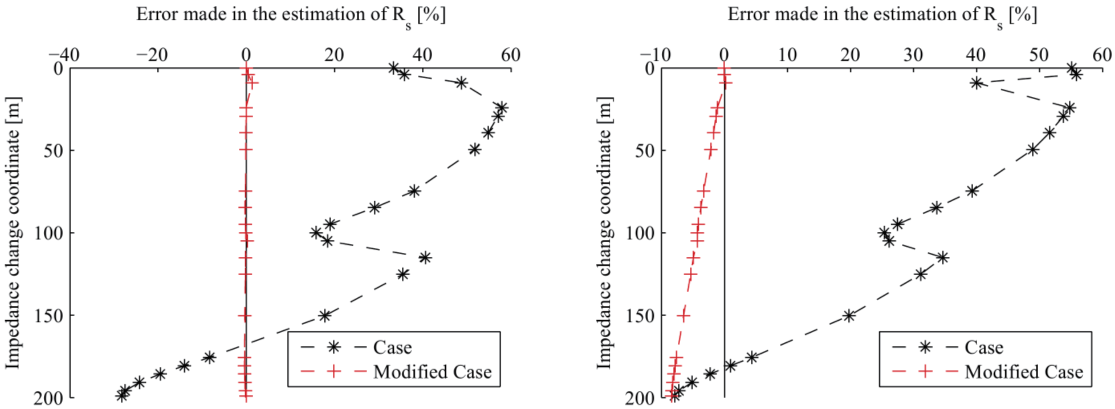
First of all, the two methods were compared on a pile of constant impedance (i=1). The results of the two methods were identical.

Then, one impedance change was imposed on the pile. The results are given as functions of the coordinate of the impedance change in Fig. 7 for $j_c = 0$. When only base resistance is allocated to the pile ($\eta_F = R_{shaft}/(R_{shaft} + R_{base}) = 0$), thus respecting the assumption of the modified Case Method), the error committed by the modified Case Method is less than 1%. In comparison, the error generated by the original Case Method ranges from -30% to +60% (Fig. 7a).

Fig. 7b shows the results for a pile with 5 MN of toe resistance and 5 MN of evenly distributed shaft friction ($\eta_F = 0.5$). Here again, in spite of the resistance toe lumping assumption, the modified Case Method provides a far better estimate of the static capacity than the original method.

Table 1. Specifications of the pile model and driving system.

Modulus of elasticity	E	200	GPa
Density	ρ	7850	kg/m²
Cross section	A	1000	cm²
Length	L	200	m
Pile impedance (at head)	I_0	3.962	MNs/m
	$2L/c$	79.250	ms
Stiffness of the pile cap	k	1000	MN/m
Hammer mass	M	0.100	Mkg
Time of wave max amplitude	t_0	9.600	ms



(a) (b)

Figure 7. Error of the original and modified Case Methods with respect to the depth of the impedance change, with $i = 2$, $j_c = 0$, base resistance: 5 MN and shaft resistance: (a) 0 MN, $\eta_F = 0$, (b) 5 MN spread equally across the pile length, $\eta_F = 0.5$.

7 CONCLUSION

This paper presents a modified version of the Case Method, maintaining an analytical estimation of the pile static resistance using force and velocity measurements at the pile head during a dynamic loading. The presented modified version takes into account a possible impedance change along the pile.

Even though a supplementary assumption had to be made (lumping all resistance at pile toe) to produce an analytically acceptable result for the modified Case Method, good results, in comparison with the original Case Method, were obtained in the numerical simulation.

The modified Case Method conserves the key advantage of the original Method: it remains an analytical and immediate estimation of the static resistance of the pile and can thus be used directly after each hammer blow.

However, both methods require radical assumptions to produce their results. Two of them are subject to question: all damping effects are lumped into the Case damping constant j_c and the downward movement of the pile assumption may be a poor bet for long piles.

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