

# NUMERICAL MODELING OF CYLINDRICAL CAVITY EXPANSION IN ROCK MASS BASED ON THE HOEK-BROWN YIELD CRITERION

## MODELISATION NUMERIQUE DE L'EXPANSION D'UNE CAVITE CYLINDRIQUE DANS UNE MASSE ROCHEUSE EN UTILISANT LE CRITERE DE PLASTICITE DE HOEK-BROWN

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**ABSTRACT** - The expansion of a cylindrical cavity in a Hoek-Brown (H-B) rock mass is numerically modeled using the finite element method. Two yield criteria are considered for the rock mass, namely the classical Hoek-Brown criterion and a power form of the extended Drucker-Prager (EDP) criterion. The findings from this study are compared to experimental results from pressuremeter tests. A good match of a simulated PMT curve is found with actual PMT data pits collected in St-Peter Sandstone.

**RÉSUMÉ** - L'expansion d'une cavité cylindrique dans une masse rocheuse est numériquement modélisée en utilisant la méthode des éléments finis. Deux critères de plasticité sont considérés pour la masse rocheuse, à savoir le critère classique de Hoek-Brown (H-B) et le critère de Drucker Prager (EDP) à forme de puissance. Les résultats de cette étude sont comparables aux résultats issus des essais pressiométriques réalisés dans les formations rocheuses de St-Peter Sandstone.

### 1. Introduction

Cavity expansion approximations are often used to study penetration phenomena. This approach has several applications in geotechnical engineering such as the stability of circular tunnels, and the penetration of piles during installation. The prediction of toe and shaft resistance of piles during driving in soils remains a challenging geotechnical problem because pile installation involves large strains. Many authors e.g. Vesic (1972), Carter et al (1986), and Randolph et al (1994) have modeled the behavior of driven piles in soils based on cavity expansion theory.

Tunneling in rock has stimulated other analytical studies: Carranza-Torres (1998) derived an analytical solution for a Tresca yield criterion; Wang et Yin (2011) developed a closed-form solution for spherical cavity collapse in a brittle plastic infinite medium where both of Mohr-Coulomb and Hoek-Brown yield criteria were considered.

### 2. Rock mass modeling

#### 2.1. Hoek-Brown failure criterion

The Hoek-Brown failure criterion is widely accepted for rock masses and has been applied in a large number of projects around the world. Hoek and Brown (1980, 1988) introduced their failure criterion in with a view to provide input data to analyses required for the design of underground excavations in hard rock. The criterion was derived from a combination of results of research on the brittle failure of intact rock conducted by Hoek and on model studies of jointed rock mass behaviour conducted by Brown.

The criterion started from the properties of intact rock, incorporating factors to reduce these properties on the basis of the characteristics of joints affecting the rock mass. It can be expressed as follows:

$$\sigma_1 = \sigma_3 + \sigma_{ci} \left( m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^\alpha \quad (1)$$

where

- $\sigma_1$  and  $\sigma_3$  are respectively the major and minor effective principal compressive stresses
- $\sigma_{ci}$  is the unconfined compressive strength of the intact rock
- $m_b$  is the reduced value of the material constant  $m_i$
- $m_i$ ,  $\alpha$ , and  $s$  are material constants ( $s=1$  for the case of intact rock) which can be expressed as functions of the geotechnical strength index (GSI) and the disturbance factor (D) as follows:

Figure 1 provides a graphical representation of equation 1, showing the influence of the GSI for different values of the disturbance factor D.

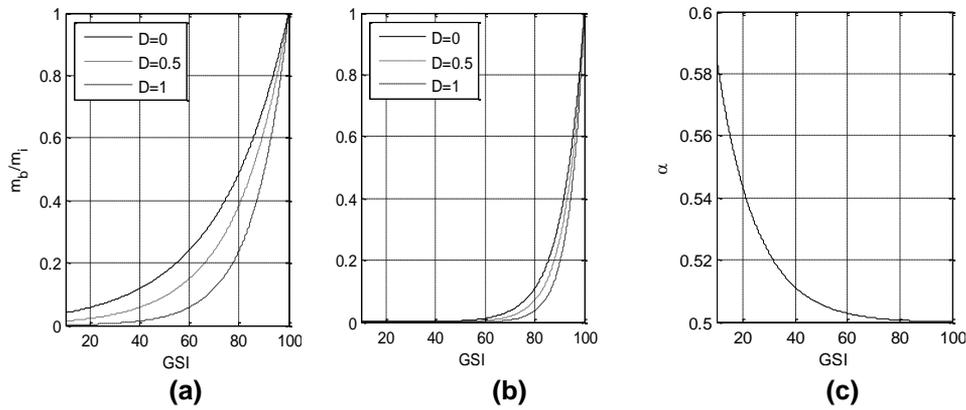


Figure 1. Hoek- Brown parameters as functions of GSI: (a)  $m_i$ , (b)  $s$ , (c)  $\alpha$

Using a cylindrical coordinate system for axi-symmetrical situation such as expansion of a cylindrical cavity, the major principal stress is the radial stress  $\sigma_r$  while the minor principal stress is the circumferential stress  $\sigma_\theta$ . Equation 1 becomes:

$$\sigma_r = \sigma_\theta + \sigma_{ci} \left( m_b \frac{\sigma_\theta}{\sigma_{ci}} + s \right) \quad (2)$$

## 2.2. Extended Drucker-Prager criterion: power form

The extended Drucker-Prager (EDP) yield criterion, available as a “material” in the ABAQUS® material library, has been found appropriate to emulate a Hoek-Brown type criterion. This material can be used for both explicit (ABAQUS/Explicit) and implicit (ABAQUS/Standard) analysis and it is typically used for granular medium such as soils and for rocks. In particular, the EDP yield criterion can be expressed under a power form as follows:

$$F(p, q) = aq^b - p - p_t \quad (3)$$

where  $p$  and  $q$  are respectively the equivalent pressure (or mean stress) and the Von Mises equivalent stress, defined as follows:

$$p = \frac{1}{3} \text{trace}(\underline{\underline{\sigma}}) \quad q = \sqrt{\frac{3}{2} \underline{\underline{S}} : \underline{\underline{S}}} \quad (4)$$

where  $\underline{\underline{\sigma}}$  and  $\underline{\underline{S}}$  are respectively the Cauchy and deviator stress tensors.

The yield criterion of the power form of the EDP in the  $p$ - $q$  plane is depicted in the following figure evidencing  $-p_t$  is the isotropic tensile limit of the material.

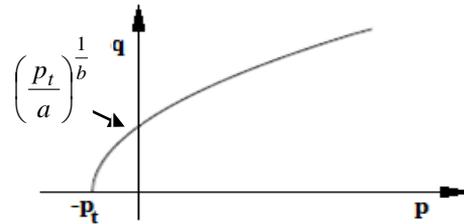


Figure 2. Yield criterion for power Drucker-Prager model (After ABAQUS [2013])

Assuming that  $\sigma_1$ ,  $\sigma_2$ , and  $\sigma_3$  are respectively the major, intermediate, and the minor principal stresses. When  $\sigma_2 = \sigma_3$ , we have the following relationships under triaxial compression ( $\sigma_2 = \sigma_3$ )

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_3 & 0 & 0 \\ 0 & \sigma_3 & 0 \\ 0 & 0 & \sigma_1 \end{bmatrix} \quad \underline{\underline{S}} = \frac{1}{3} \begin{bmatrix} \sigma_3 - \sigma_1 & 0 & 0 \\ 0 & \sigma_3 - \sigma_1 & 0 \\ 0 & 0 & 2\sigma_1 - 2\sigma_3 \end{bmatrix} \quad (5)$$

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3) \quad q = \sigma_1 - \sigma_3 \quad (6)$$

Parameters of the EDP criterion can be chosen to produce a yield function approximating a given H-B yield criterion. An explicit determination of parameters  $a$ ,  $b$ , and  $p_t$  can even be developed provided the two yield curves coincide at three selected points. For the purpose of the present development, the following stress points (A, B, and C) have been selected to that end:

Point A:

$$p_A = -\frac{s\sigma_{ci}}{m_b} \quad \& \quad q_A = 0 \quad (7)$$

Point B:

$$p_B = \frac{1}{3} \sigma_{ci} \cdot s^\alpha \quad \& \quad q_B = \sigma_{ci} \cdot s^\alpha \quad (8)$$

Point C:

$$p_C = \frac{\sigma_{ci}}{24} \left( m_b + \sqrt{m_b^2 + 144 \cdot s} \right) \quad \& \quad q_C = \frac{\sigma_{ci}}{12} \left( m_b + \sqrt{m_b^2 + 144 \cdot s} \right) \quad (9)$$

Based on equations 7, 8, and 9, parameters of the EDP criterion can be directly obtained from the following expressions:

$$p_t = \frac{s\sigma_{ci}}{m_b} \quad b = \frac{\ln\left(\frac{p_B + p_t}{p_C + p_t}\right)}{\ln\left(\frac{q_B}{q_C}\right)} \quad a = \frac{p_B + p_t}{q_B^b} \quad (10)$$

Figure 3 compares in the p-q plane the yield surfaces for the H-B and the EDP yield criteria for St-Peter Sandstone having the following H-B parameters. It is noted that the H-B parameters of the considered rock were estimated using the RockLab software.

Table I. H-B and EDP parameters for St-Peter sandstone derived from Dittes and Labuz (2002)

$\sigma_{ci} = 52 \text{ MPa}$	$\sigma_c = 1.8 \text{ MPa}$	$a = 0.035$
$\text{GSI} = 40$	$\sigma_t = 0.033 \text{ MPa}$	$b = 1.16$
$m_i = 17$	$m_b = 2$	$p_t = 0.033 \text{ MPa}$
$D=0$	$s=0.0013$	
	$\alpha=0.5$	
	$E=4 \text{ GPa}$	
	$\nu=0.33$	
H-B basic parameters	H-B derived parameters	EDP parameters

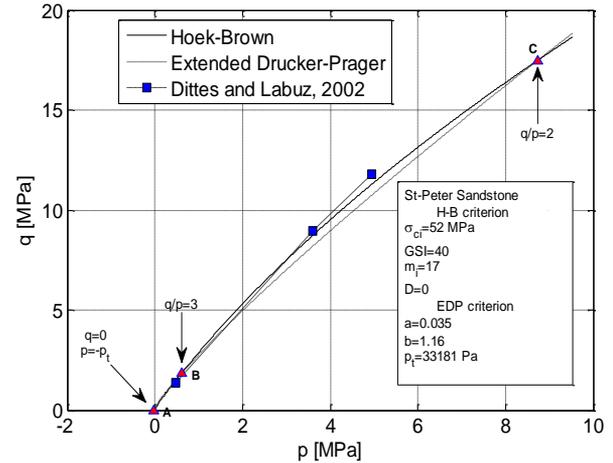


Figure 3. Yield criterion in p-q plane for both H-B and EDP

It can be noted that the agreement appears satisfactory over a stress  $p$  ranging from  $-\sigma_t$  to  $100.\sigma_t$ .

### 3. Numerical triaxial tests

In order to assess the validity of the proposed EDP parameters, a set of numerical tri-axial tests have been simulated using ABAQUS software on a specimen ( $D=100 \text{ mm}$ ,  $L=200 \text{ mm}$ ) of St-Peter Sandstone. The triaxial test consisted in applying a confining pressure ( $\sigma_3$ ) according to the x-direction, and displacement ( $\delta$ ) according to the y-direction as shown by figure 4.

Figure 5 shows the variation of the major principal ( $\sigma_1$ ) stress as a function of the axial strain ( $\epsilon_1$ ) for different confining pressures

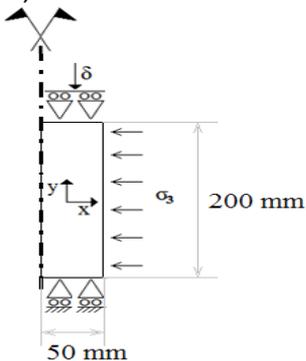


Figure 4. Triaxial test on a St-Peter Sandstone specimen

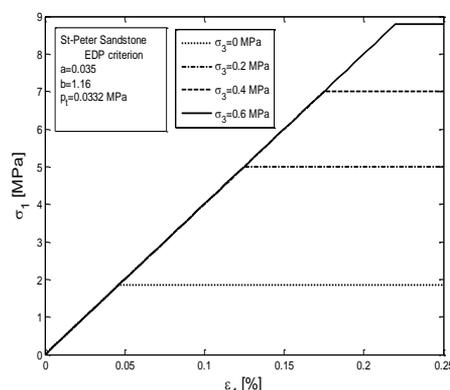


Figure 5. Major principal stress as function of axial strain for different confining pressures

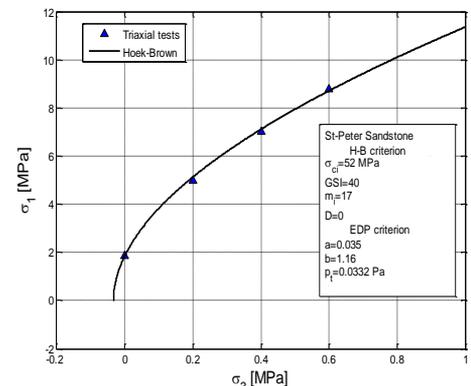


Figure 6. Comparison between the yield points obtained by numerical triaxial compression tests and the theoretical H-B yield criterion in the  $\sigma_1$ - $\sigma_3$  ( $=\sigma_2$ ) plane

## 4. Cylindrical cavity expansion

### 4.1. Problem statement

The wall of an infinitely long cylindrical cavity with radius  $r_0$  in a homogeneous infinite isotropic rock mass is subjected to an internal pressure  $P$ . The medium is initially isotropic and subjected to a hydrostatic stress  $\sigma_0$ . The problem geometry and boundary conditions are depicted in figure 7 where a cylindrical coordinate system is adopted. Because of axial symmetry, the problem is reduced to a plane strain problem that can be fully depicted using a single radial coordinate 'r'.

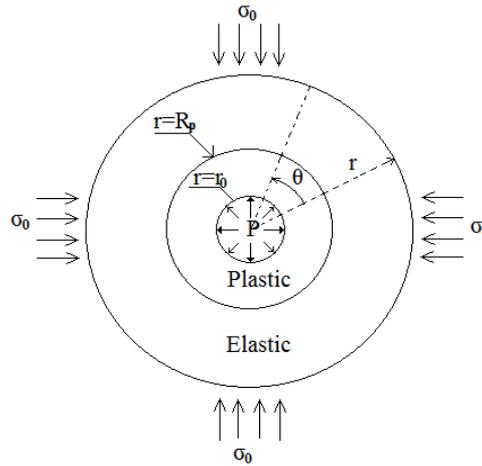


Figure 7. Cylindrical cavity expansion in rock mass

The plastic flow rule of the rock mass is associated to the H-B yield criterion as described in equation 2. The purpose of the simulation is to portray the response of the cavity to an increase of the internal pressure  $P$ . Two particular pressures will be assessed:

- $P_{\text{yield}}$ , identifying the yielding pressure
- $P_1$ , identifying the doubling of the initial volume of the cavity

### 4.2. Analytical expression of yield pressure

In order to assess the yielding pressure  $P_{\text{Yield}}$ , let's consider an infinite homogeneous and isotropic elastic rock medium. The elastic response of a cylindrical cavity subjected to an internal pressure  $P$  is given in terms of stresses by the following equations:

$$\sigma_r = \sigma_0 + (P - \sigma_0) \left( \frac{r_0}{r} \right)^2 \quad (11a)$$

$$\sigma_\theta = \sigma_0 - (P - \sigma_0) \left( \frac{r_0}{r} \right)^2 \quad (11b)$$

It can be noted that the  $\sigma_r$  and  $\sigma_\theta$  profiles are independent from the elastic parameters ( $E$ ,  $\nu$ ). The yield pressure can be expressed by substituting equations 11a, and 11b into equation 2

$$P_{\text{Yield}} = \sigma_0 + \frac{\sigma_{ci}}{8} \left( -m_b + \sqrt{m_b^2 + 16s + 16m_b \frac{\sigma_0}{\sigma_{ci}}} \right) \quad (12)$$

It can be noted also that the yield pressure is independent from the radius of the cavity and from any assumed elastic parameters provided the medium is homogeneous and isotropic.

### 4.3. Numerical modeling of the cylindrical cavity

The geometry ( $r_0=38.1\text{mm}$ ) and boundary conditions of the axi-symmetric plane strain model for the cylindrical cavity expansion problem are shown in figure 8. The rock layer is depicted into two zones: the near field zone (up to  $200 r_0$ ) is modeled by continuum finite elements (CAX4) while the far field is modeled by continuum infinite elements (CINAX4) since the theoretical solution of the cylindrical cavity expansion is based on infinite continuum. The EDP yield criterion is governed by the parameters listed in Table I.

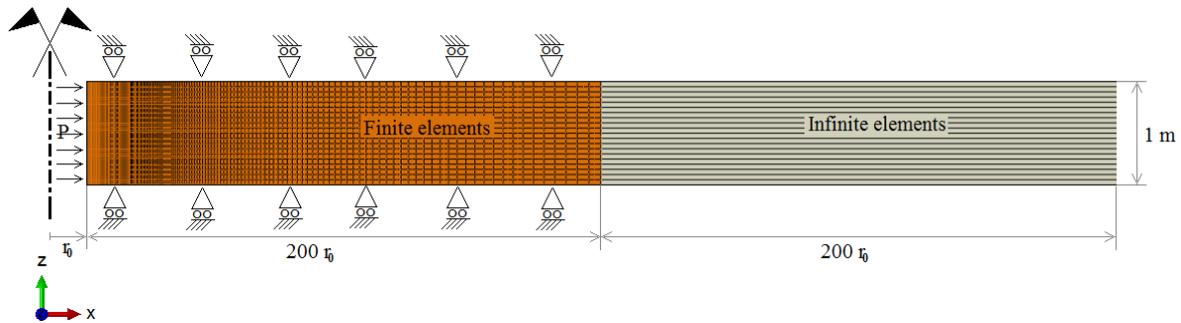


Figure 8. Numerical modeling of the cylindrical cavity expansion

## 5. Results and analysis

### 5.1. Numerical yielding pressure

Figure 9 shows the variation of the radial plastic strain  $\varepsilon_r^p$  at the cavity wall ( $r=r_0$ ) as a function of the pressure applied at the cavity wall. Figure 9 evidences a yield pressure of a 3.1 MPa which corresponds to that found using the analytical approach described by equation 12. During this simulation, the horizontal ambient stress  $\sigma_{\infty, r}$  is equal to 1.5 MPa.

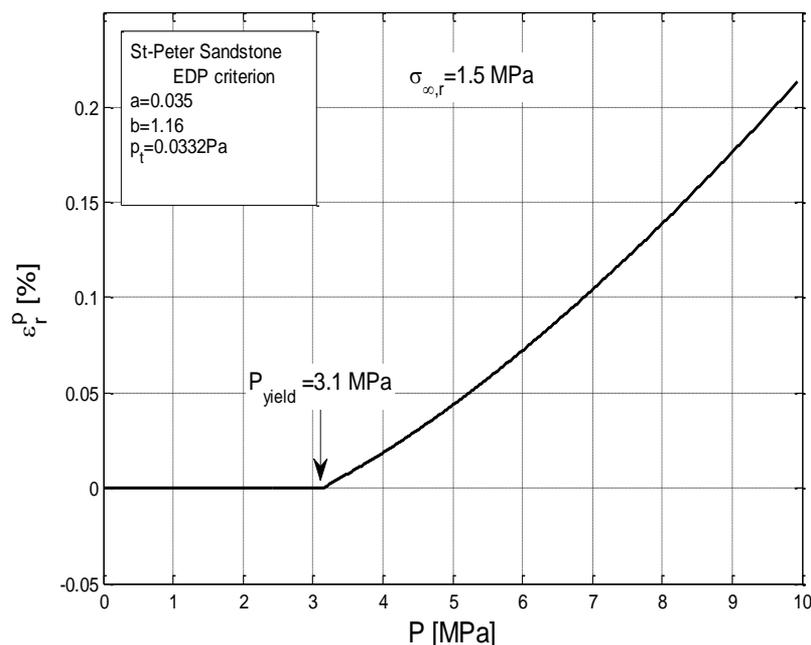


Figure 9. Radial plastic strain at the cavity wall Vs applied pressure (ABAQUS)

## 5.2. Stresses assessment

A comparison of the numerical results with the analytical elastic solution given by equations 11a, and 11b, shows that both numerical and elastic analytical approaches give the same results in terms of radial and circumferential stresses as long as the applied cavity pressure  $P$  remains lower than  $P_{yield}$ . Figure 10 (a) shows that match for  $P=2.5$  MPa while figure 10 (b) shows its deviations for  $P=25$  MPa.

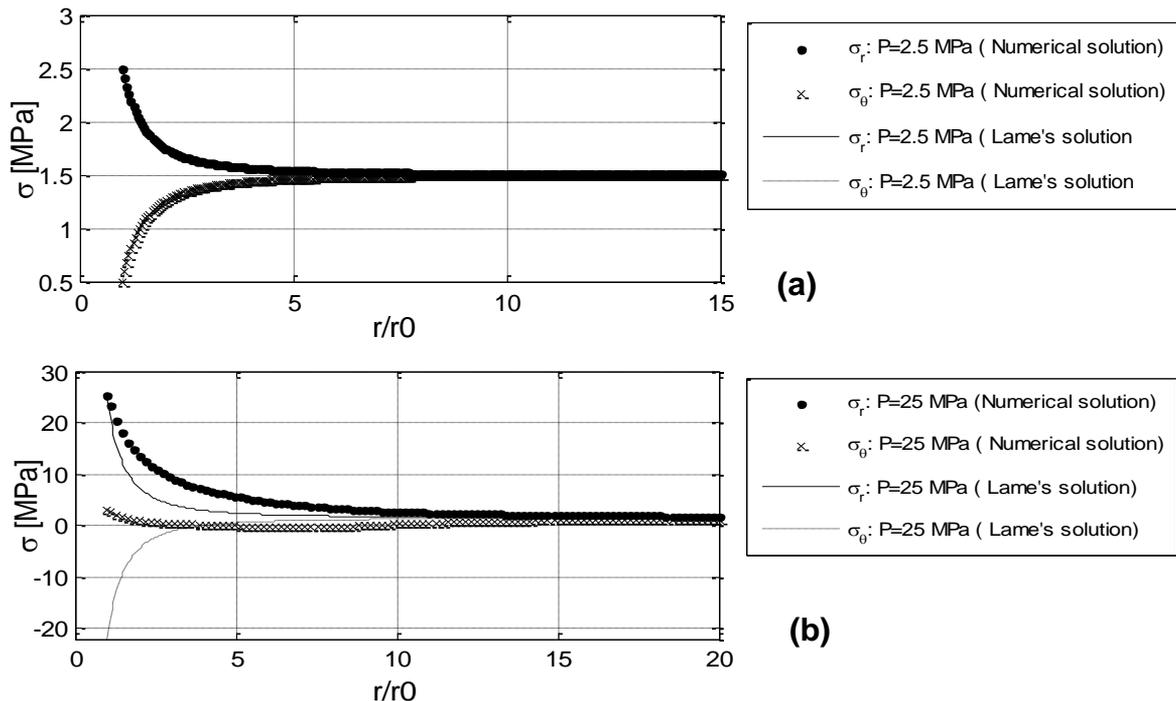


Figure 10. Comparison of radial and circumferential stresses with elastic analytical solution: (a)  $P=2.5$  MPa  $< P_{yield}$ , (b)  $P=25$  MPa  $> P_{yield}$

## 5.3. Comparison with pressuremeter tests:

Results of in situ pressuremeter tests performed in St. Peter sandstone using a probe having a diameter 76.2 mm and a length of 984 mm have also been reported by Dittes and Labuz (2002). Figure 11 shows a comparison between their experimental pressuremeter test results and the numerical modeling of the expansion of the PMT cylindrical cavity in an EDP rock mass. The simulated  $P$ - $\varepsilon_\theta$  curve matches the experimental data pits closely

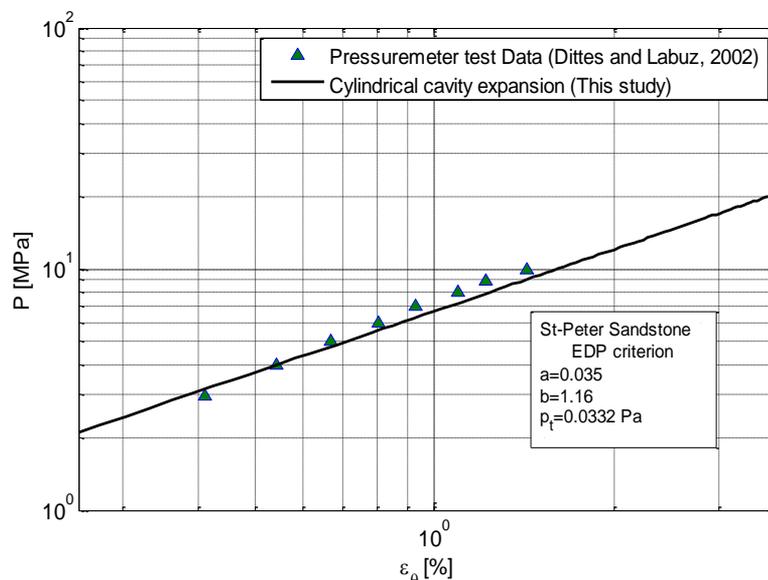


Figure 11. Comparison between pressuremeter tests and present study

## 6. Conclusion

A numerical model for cylindrical cavity expansion problems in rock masses has been presented. Two yield criteria were considered for the rock mass, i.e. the Hoek-Brown yield criterion and Extended Drucker-Prager yield criterion

The generalized Hoek-Brown yield criterion is described by four independent parameters ( $\sigma_{ci}$ , GSI,  $m_i$ , D). In the case of undisturbed rocks (D=0), the EDP yield criterion parameters can be explicitly chosen to produce a yield function approximating the H-B yield criterion. Numerical triaxial tests were performed and results show a good agreement between the yield compressive stresses given by H-B and EDP criteria.

Numerical modeling of a cylindrical cavity expansion in a rock mass using the EDP criterion was performed. Comparison of the numerical simulation and Lamé's elastic solution (elastic expansion of a cylindrical cavity) gave the same results in terms of stresses and yield pressure. A reasonable level of agreement was also found when comparing numerical simulations to results obtained by pressuremeter tests performed in St. Peter sandstone.

## 7. References

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